Structure of Exogenous Processes, Aggregate Fluctuations, and Asset Price Movements: Bayesian Estimation of a DSGE Model*

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Abstract

This study develops a dynamic and stochastic general equilibrium model and exploits Bayesian inference methods to investigate the major driving forces of fluctuations in aggregate variables and asset prices. Taking into account the possibility that the growth of total factor productivity, labor augmenting technology and investment-specific technology might consist of permanent and transitory components, this study considers a baseline and three alternative structures of the exogenous processes. We find that the identification of the major sources of aggregate fluctuations hinges critically upon researchers' assumptions on the structure of the exogenous processes. Bayesian model comparison indicates that previous studies might have overlooked the persistent change in investment-specific technology growth, and thus underestimate its importance to driving the business cycles. Using the structure of the exogenous processes that is mostly favored by the data, we find that investment-specific technology shocks contribute a significantly large fraction of the short-run and the long-run fluctuations in output growth, investment growth, and the share of total market values in output. In addition, shocks to the permanent and the transitory components of investment-specific technology growth also explain a majority of the predicted error variance of consumption-output ratio and hours in the long-run. In contrast, labor augmenting technology shocks, preference shocks and government spending shocks are only important contributors to changes in hours, consumption and government expenditures, respectively, in the very short-run.

Keywords: DSGE; Aggregate Fluctuations; Asset Prices; Bayesian Inference.
JEL Classification: C11, C51, E13, E32

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1 Introduction

Over the past few decades, a large body of the real business cycle literature has attempted to identify the major sources of aggregate fluctuations. Exploiting the Dynamic and Stochastic General Equilibrium (DSGE) framework and taking into account various aspects of the real rigidities in the economy, recent business cycle studies have achieved remarkable improvements along the dimension of reproducing the key stylized facts of the post war U.S. data in the simulated environment. However, the consensus of the key driving forces of the business cycles has been barely reached among economists, and the debate remains.

Before late 1980s, the conventional wisdom of the business cycle literature suggests that fluctuations in aggregate variables are overwhelmingly due to macroeconomic innovations to total factor productivity and labor-augmenting technology. Greenwood, Hercowitz and Huffman (1988), however, highlight the importance of investment-specific technology shocks to driving the business cycles. The view that investment-specific technology shocks play a non-negligible role in accounting for the business cycle facts is further supported by Greenwood, Hercowitz and Krusell (1997) and Fisher (2006), but challenged later by Justiniano, Primiceri, and Tambalotti (2011). In particular, Justiniano, Primiceri, and Tambalotti (2011) consider two investment innovations that potentially affect capital accumulation at the aggregate level. Those are, the standard investment-specific technology shocks that govern the efficiency in transforming consumption goods into investment goods, and the shocks to the marginal efficiency of investment which regulate the transformation rate of investment goods into capital. According to their empirical findings, the second shocks explain over 50% of the predicted error variance of output, hours and investment, whereas the standard investment-specific technology shocks play no role in driving the business cycles. Along the line of effort that disentangles the role of investment-related innovations, Schmitt-Grohé and Uribe (2012) take into account news (or anticipated) shocks under a medium-scaled DSGE framework where households have Jaimovich-Rebelo preferences, and provide empirical evidence largely consistent with that of Justiniano, Primiceri, and Tambalotti (2011).

So as to fully understand the causes and the consequences of the business cycles, it seems natural to carefully investigate the potential sources leading to the aforementioned empirical discrepancies. Inarguably, inconsistencies in model implications have to be accounted for by varying specifications on the underlying analytical framework and the identification procedures adopted for the quantitative practice. Existing studies have unambiguously demonstrated that the identification of the major contributors to business fluctuations is largely affected by the data incorporated in the observable vector. For instance, Avdjiev (2009) and Schmitt-Grohé and Uribe (2012) show that estimation of DSGE models with and without asset market data can yield remarkably distinct model-implied aggregate dynamics. In particular, Schmitt-Grohé and Uribe (2012) report that investment-specific technology shocks play trivial role when the data on the relative price of investment is incorporated into the observable vector, but become a significant contributor to aggregate fluctuations once excluded.

For the extent to which the identification of the major sources of business fluctuations is contingent upon the analytical framework adopted for the quantitative analysis, the existing literature provides quite limited guidance. The rarity of work along that line of effort is not necessarily incomprehensible. First, each study investigates a uniquely specific topic, and has its own focuses and concerns. Therefore, the assumption made on the modeled economy is completely up to the underlying research purposes, and hence, needs to be respected. In addition, comparing empirical evidence across analytical frameworks with fundamentally distinct features seems not a feasible research avenue for reconciling the inconsistencies between previous studies, because the impact of fundamental differences in modeling choices (such as preference specifications, incorporation and exclusion of certain sectors, and so forth) on identifying the business cycle drivers seems almost not quantitatively traceable.

Given these concerns, this study investigates the major business cycle contributors and attempts to shed light on the potential sources of the documented empirical discrepancies from a novel perspective. To be specific, conditional on a proposed economy capable of replicating the key features of its actual counterpart, this study constructs a set of competing alternatives through varying exclusively the assumptions on the data generating processes of the exogenous variables, and seeks to assess whether the model implications on the key business cycle drivers are sensitive to these small changes in model specifications when the remaining features of the modeled economy are well retained. We restrict our attention to the structure of stochastic processes taken by the exogenous variables not only due to its apparent tractability, but also because its importance to understanding the business cycle phenomena has been constantly overlooked. Previous studies in the business cycle literature usually assume that the processes of the exogenously determined variables take stationary auto-regressive forms. This conventional specification, however, normally does a poor job in capturing the behavior of inpersistent variables whose long-run movements are otherwise persistent. Given that the true data generating processes of the exogenous variables are unobservable, precluding the possible existence of persistent long-run components embedded can potentially lead to remarkably distinct, if not jeopardized, model implications. For instance, Bansal and Yaron (2004) demonstrate that, under their partial equilibrium framework where households have Epstein-Zin recursive preferences, several key asset pricing puzzles can be largely resolved when consumption growth is specified as a white noise process consisting of a small long-run component.

In this paper, we develop a neoclassical growth model under which aggregate dynamics and asset price movements are driven by innovations to technology growth, preferences and government expenditures. Our theoretical framework is closely related to those of Schmitt-Grohé and Uribe (2008, 2012), and Avdjiev (2009). The proposed economy is augmented with four real rigidities, namely variable capacity utilization, capital adjustment costs, and internal habit formation in consumption and leisure. For each model specification, we exploit Bayesian techniques to estimate the unknow economic parameters, and then perform variance decomposition and impulse responses analysis to draw model inference. We find that the identification of the business cycle drivers hinges critically on the structure of the exogenous processes. Under the specifications where investment-specific technology growth consists of a permanent component, a vast majority of the fluctuations in output growth, investment growth, hours and total market values is accounted for by shocks to investment-specific technology growth. In contrast, when a permanent component is embedded in labor-augmenting technology growth but not investment-specific technology growth, the variance decomposition statistics indicate that the major contributors to business cycles are shocks to total factor productivity and labor-augmenting technology growth. While consistent with the findings of Schmitt-Grohé and Uribe (2012), this specification, however, is not supported by the U.S. data. Bayesian odd ratio test indicates that previous studies might have overlooked the persistence in investment-specific technology growth, and hence, underestimate its importance to explaining the business cycle facts.

The rest of this paper is organized as follows. We introduce our theoretical framework in section 2. Section 3 describes the baseline and the three alternative specifications on the structure of the exogenous processes. Estimation procedure, calibration choices and parameter estimates are presented in section 4. Model inference based on variance decomposition and Bayesian model comparison is discussed in section 5. Section 6 reports the impulse responses analysis. And section 7 concludes.

2 The Model

In this study, we consider an economy with a stand-in representative household, whose preference over consumption and leisure is given by

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} e^{\xi_{t+i}} \left\{ \frac{\left[\left(C_{t+i} - b_c C_{t-1+i} \right) \left(L_{t+i} - b_l L_{t-1+i} \right)^{\chi} \right]^{1-\gamma} - 1}{1-\gamma} \right\} ,$$

where $\beta \in (0,\ 1)$ denotes household's subjective discount factor; ξ_t represents the stochastic preference shock at time t; C_t and L_t denote the consumption goods and leisure comsumed by the household, respectively; $b_c \in [0,\ 1]$ and $b_l \in [0,\ 1]$ capture the degree of internal habit formation in consumption and leisure, respectively; χ governs the Frisch elasticity of labor supply; and γ denotes the inverse of intertemporal elasticity of substitution (IES).

Let H_t denote hours worked by the representative household. Normalizing total hours to unity yields $L_t + H_t = 1$ for all t. Rewriting the life-time utility function in C_t and H_t, we have

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} e^{\xi_{t+i}} \left\{ \frac{\left[\left(C_{t+i} - b_c C_{t-1+i} \right) \left(\tau + b_l H_{t-1+i} - H_{t+i} \right)^{\chi} \right]^{1-\gamma} - 1}{1-\gamma} \right\} , \tag{2.1}$$

where $\tau = 1 - b_l$ is constant.

Assume that the representative household is the owner of physical capital. The law of motion of capital is specified as follows,

$$K_{t+1} = \left[1 - \delta \left(u_t \right) \right] K_t + e^{z_t^I} I_t \left[1 - \Phi \left(\frac{I_t}{I_{t-1}} \right) \right] , \tag{2.2}$$

where K_t, predetermined in period $t-1$, represents the capital stock in period t; I_t denotes the level of gross investment; z_t^I is interpreted as a transitory investment-specific productivity disturbance that governs the efficiency in transforming investment goods into physical capital; and u_t denotes the time-varying rate of capital utilization. By definition, the effective amount of capital available for firms' production in each period is given by $u_t K_t$.We assume that the depreciation rate of capital $\delta(\cdot)$ is an increasing and convex function of u_t, which is given by

$$\delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2 \ . \tag{2.3}$$

In equation (2.3), δ_0 refers to the non-stochastic steady-state depreciation rate; δ_1 is determined by the steady-state equilibrium conditions when u_t is normalized to unity; and δ_2 regulates the sensitivity of capital utilization to variation in the rental rate of capital.[1]

In equation (2.2), the function $\Phi(\cdot)$ captures the investment adjustment cost, which is specified as follows,

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \mu^i\right)^2 \ , \tag{2.4}$$

where κ is constant; and μ^i is the steady state growth rate of investment.

In addition, we assume that the representative household's sequential budget constraint is given by

$$C_t + A_t I_t + \Gamma_t = W_t H_t + r_t u_t K_t + P_t \ . \tag{2.5}$$

In equation (2.5), A_t denotes the non-stationary stochastic productivity factor which affects the technology rate of transforming consumption goods into investment goods; W_t and r_t denote the competitive wage of labor supply and the rental rate of capital, respectively; P_t represents the profit that the household collects from the firm; and Γ_t is the lump-sum tax paid to the government.[2] Given A_t's non-stationarity, we define the gross growth rate of A_t as

$$\mu_t^a = \frac{A_t}{A_{t-1}} \ , \tag{2.6}$$

and assume that μ_t^a follows a stationary stochastic process with steady-state value equal to μ^a.

According to the above model specification, household's optimization problem is choosing a sequence of $\{C_t,\ H_t,\ I_t,\ K_{t+1},\ u_t\}_{t=0}^{\infty}$ to maximize equation (2.1), subject to equations (2.2) and (2.5), and taking as given the stochastic processes $\{\xi_t,\ Z_t^I,\ A_t,\ W_t,\ P_t\}_{t=0}^{\infty}$ and the initial condition C_{-1}, I_{-1} and K_0.

For the production side of the modeled economy, we assume that the representative firm uses effective capital stock and labor as inputs, and its production function takes the following Cobb-Douglas form:

$$Y_t = e^{z_t} F(u_t K_t,\ X_t H_t) = e^{z_t} (u_t K_t)^{\alpha} (X_t H_t)^{1-\alpha} \ , \tag{2.7}$$

[1] For details regarding how δ_1 is determined, please refer to Appendix B.

[2] We assume that the representative household owns the firm.

where Y_t denotes output; z_t is total factor productivity shock; and X_t is a non-stationary labor-augmented technology shock. We further define the gross growth rate of X_t as

$$\mu_t^x = \frac{X_t}{X_{t-1}} \, . \tag{2.8}$$

We assume that μ_t^x follows a stationary process with steady-state value μ^x.

We assume that the government in each period consumes an exogenous amount of resource G_t, which is financed by levying lump-sum taxes. Inspired by Schmitt-Grohé and Uribe (2012), we assume that G_t has a stochastic trend component, X_t^G, which is specified as follows,

$$X_t^G = \left(X_{t-1}^G\right)^{\rho_{xg}} \left(X_{t-1}^Y\right)^{1-\rho_{xg}} \, , \tag{2.9}$$

where ρ_{xg} is a constant. According to our specification, first, government expenditures and output are allowed to be cointegrated, which ensures that the share of government expenditures in output is stationary. Second, the trend of government consumption X_t^G is potentially smoother than the trend of output, namely X_t^Y, and the degree of smoothness is regulated by ρ_{xg}. In addition, given that X_t^G is determined by the information available in period $t-1$, equation (2.9) permits lagged responses of X_t^G to contemporaneous changes in the trend component of aggregate output. These features are consistent with the stylized facts of the post war U.S. data.

It is straightforward to show that the aggregate resource constraint takes the following form:

$$C_t + A_t I_t + G_t = Y_t \, . \tag{2.10}$$

Given that the modeled economy is free of distortions, solving the competitive equilibrium allocation is equivalent to solving a social planner's problem. Let $\mathit{\Lambda}_t$ and $\mathit{\Lambda}_t Q_t$ denote the Lagrangian multipliers on equations (2.10) and (2.2). Then, the corresponding social planner's problem can be formulated as

$$\max_{C_{t+i},\, H_{t+i},,\, I_{t+i},\, K_{t+1+i},\, u_{t+i}} \quad E_t \sum_{i=0}^{\infty} \beta^{t+i} \Bigg\{ e^{\xi_{t+i}} \frac{\left[(C_{t+i} - b_c C_{t-1+i})(\tau + b_l H_{t-1+i} - H_{t+i})^{\chi}\right]^{1-\gamma} - 1}{1-\gamma}$$

$$+ \mathit{\Lambda}_{t+i} \left[e^{z_{t+i}} \left(u_{t+i} K_{t+i}\right)^{\alpha} \left(X_{t+i} H_{t+i}\right)^{1-\alpha} - \left(C_{t+i} + A_{t+i} I_{t+i} + G_{t+i}\right) \right]$$

$$+ \mathit{\Lambda}_{t+i} Q_{t+i} \left[\left[1 - \delta\left(u_{t+i}\right)\right] K_{t+i} + e^{z_{t+i}^I} I_{t+i} \left[1 - \Phi\left(\frac{I_{t+i}}{I_{t-1+i}}\right)\right] - K_{t+1+i} \right] \Bigg\} \, .$$

Taking as given the set of exogenous stochastic processes $\{\xi_t,\ z_t,\ A_t,\ X_t,\ z_t^I,,\ G_t\}_{t=0}^{\infty}$ and the initial values of C_{-1}, I_{-1} and K_0, we solve the optimization problem faced by the social planner, and obtain a competitive equilibrium where the set of stochastic processes $\{C_t,\ H_t,\ I_t,\ K_{t+1},\ u_t,\ Y_t,\ \mathit{\Lambda}_t,\ Q_t\}_{t=0}^{\infty}$ satisfies equations (2.2), (2.7), (2.10), as well as the following first order conditions:

$$
\begin{aligned}
\Lambda_t \quad = & \quad e^{\xi_t}\left(C_t - b_c C_{t-1}\right)^{-\gamma}\left(\tau + b_l H_{t-1} - H_t\right)^{\chi(1-\gamma)} \\
& -\beta b_c E_t\left\{e^{\xi_{t+1}}\left(C_{t+1} - b_c C_t\right)^{1-\gamma}\left(\tau + b_l H_t - H_{t+1}\right)^{\chi(1-\gamma)}\right\} ;
\end{aligned} \tag{2.11}
$$

$$
\begin{aligned}
\Lambda_t e^{z_t}(1-\alpha)\left(u_t K_t\right)^{\alpha}\left(X_t H_t\right)^{-\alpha} \quad = & \quad \chi e^{\xi_t}\left(C_t - b_c C_{t-1}\right)^{-\gamma}\left(\tau + b_l H_{t-1} - H_t\right)^{\chi(1-\gamma)-1} \\
& -\chi\beta E_t\left\{e^{\xi_{t+1}}\left(C_{t+1} - b_c C_t\right)^{-\gamma}\left(\tau + b_l H_t - H_{t+1}\right)^{\chi(1-\gamma)-1}\right\} ;
\end{aligned} \tag{2.12}
$$

$$
\begin{aligned}
A_t \Lambda_t \quad = & \quad \Lambda_t Q_t e^{z_t^I}\left[1 - \Phi\left(\tfrac{I_t}{I_{t-1}}\right) - \left(\tfrac{I_t}{I_{t-1}}\right)\Phi'\left(\tfrac{I_t}{I_{t-1}}\right)\right] \\
& +\beta E_t\left\{\Lambda_{t+1} Q_{t+1} e^{z_{t+1}^I}\left(\tfrac{I_{t+1}}{I_t}\right)^2 \Phi'\left(\tfrac{I_{t+1}}{I_t}\right)\right\} ;
\end{aligned} \tag{2.13}
$$

$$
\Lambda_t Q_t \quad = \quad \beta E_t\left\{\Lambda_{t+1}\left[e^{z_{t+1}}\alpha u_{t+1}\left(u_{t+1}K_{t+1}\right)^{\alpha-1}\left(X_{t+1}H_{t+!}\right)^{1-\alpha} + Q_{t+1}\left(1 - \delta\left(u_{t+1}\right)\right)\right]\right\} ; \tag{2.14}
$$

$$
e^{z_t}\alpha u_t\left(u_t K_t\right)^{\alpha-1}\left(X_t H_t\right)^{1-\alpha} \quad = \quad Q_t \delta'\left(u_t\right) \quad . \tag{2.15}
$$

Notice that Q_t can be interpreted as the marginal Tobin's q, which captures the relative price of capital stock to consumption goods.

Similar to the standard real business cycle models in the literature, the proposed model in this paper can be extended straightforwardly to capture the asset market fluctuations driven by responses of market fundamentals to macroeconomic shocks. First, firm's profit in each period is given by

$$
P_t = Y_t - W_t H_t - A_t I_t \, .
$$

Since the equilibrium wage in the competitive labor market is equal to the marginal productivity of labor, we have

$$
W_t = (1-\alpha)\, e^{z_t}\left(u_t K_t\right)^{\alpha}\left(X_t H_t\right)^{-\alpha} = (1-\alpha)\,\frac{Y_t}{H_t} \, .
$$

Therefore, firm's profit can be rewritten as

$$
P_t = \alpha Y_t - A_t I_t \, . \tag{2.16}
$$

In addition, it can be shown that the one period ahead gross risk-free rate R_t^{rf} satisfies

$$
R_t^{rf} = \frac{1}{\beta} E_t\left(\frac{\Lambda_t}{\Lambda_{t+1}}\right) \, . \tag{2.17}
$$

Then, the end-of-period firm value, V_t, satisfies the following recursive condition:

$$V_t = \beta E_t \left[\left(\frac{\Lambda_t}{\Lambda_{t+1}} \right) (V_{t+1} + P_{t+1}) \right] . \tag{2.18}$$

Therefore, in the proposed economy which consists of of the representative household, the representative firm, the government, and the asset market, the complete set of equilibrium conditions is given by equations (2.2), (2.7), (2.10), and (2.11) - (2.18). Given that these equilibrium conditions are characterized by non-stationary aggregate variables, however, a steady state does not exist. In order to obtain a stationary state-space representation of the equilibrium conditions, we transform these equilibrium conditions into their corresponding stationary form via detrending the non-stationary variables by their respective trends. Detailed discussion is provided in Appendix A.

3 Structure of Exogenous Processes

A large body of previous studies in the real business cycle literature tend to assume that the processes of exogenously determined variables take stationary auto-regressive forms. While straightforward to implement, this conventional specification overlooks the fact that those exogenous variables are usually unobserved, and thus precludes the possibility that some might be persistent in nature. Even though estimating the auto-regression coefficients of the unobservables can be informative about their transition dynamics, the auto-regression specification does not suffice to reasonably capture the behavior of inpersistent variables whose long-run movements are otherwise persistent. As suggested in Shephard and Harvey (1990), it is almost impossible to exploit finite samples to distinguish between a pure white noise process and a white noise process with a small persistent component. And neglecting the seemingly tiny difference in data generating process can potentially lead to remarkably distinct model implications. A well-known example can be borrowed from the long-run risks literature. Unlike the conventional consumption-based asset pricing studies which assume that consumption growth is pure white noise, Bansal and Yaron (2004) propose that there exists a small persistent component embedded in the consumption growth process, and find that the incorporation of long-run consumption growth helps to resolve several major asset pricing puzzles.

Due to the aforementioned concern, we choose to decompose three key exogenous variables, namely total factor productivity growth (z_t), labor-augmenting technology growth (μ_t^x) and investment-specific technology growth (μ_t^a), into permanent and transitory components. By definition, a permanent component of a given process regulates its long-run persistence, and a transitory component governs its transitory dynamics.[3] Since one of the major goals of this paper is to investigate whether the identification of the major driving forces of aggregate fluctuations is contingent upon researchers' specifications on the structure of the exogenous processes, this study considers one baseline model along with three alternatives. In addition,

[3]Similar specification can be found in Balke and Wohar (2002).

note that it is not impossible that the structure of the exogenous processes is misspecified if the unobserved true data generating process of a given variable does not contain a permanent component. Therefore, to investigate a set of competing alternatives seems particularly necessary, since it would allow us to perform model comparison (using Bayesian inference techniques) to minimize the impact of misspecification, and eventually identify the one that is mostly favored by the data.

To be specific, for the growth rate of labor-augmenting technology and investment-specific technology, we define the percentage deviation of μ_t^x and μ_t^a from their respective steady state as

$$\widehat{\mu_t^x} = log\left(\frac{\mu_t^x}{\mu^x}\right) \ ;$$

$$\widehat{\mu_t^a} = log\left(\frac{\mu_t^a}{\mu^a}\right) \ ;$$

or simply,

$$\widehat{\mu_t^j} = log\left(\frac{\mu_t^j}{\mu^j}\right) \quad for \ j = \{x, \ a\} \ .$$

In the baseline model, we assume that $\widehat{\mu_t^j}$ consists of a permanent component $\widehat{\mu_t^{j,P}}$, and a transitory component $\widehat{\mu_t^{j,T}}$.

$$\widehat{\mu_t^j} = \widehat{\mu_t^{j,P}} + \widehat{\mu_t^{j,T}} \ . \tag{3.1}$$

We further specify the processes of these two components as follows,

$$\widehat{\mu_t^{j,P}} = \phi^{j,P}\widehat{\mu_{t-1}^{j,P}} + \epsilon_t^{j,P} \ ; \tag{3.2}$$

$$\widehat{\mu_t^{j,T}} = \phi_1^{j,T}\widehat{\mu_{t-1}^{j,T}} + \phi_2^{j,T}\widehat{\mu_{t-2}^{j,T}} + \epsilon_t^{j,T} \ . \tag{3.3}$$

First, equation (3.1) captures the persistent long-run movements in $\widehat{\mu_t^j}$. To avoid introducing non-stationarity to the equation system, we assume that $\phi^{j,P}$ is close to, but stictly less than, one. Second, the transitory component $\widehat{\mu_t^{j,T}}$ is assumed to follow an $AR(2)$ process with auto-regression coefficients $\phi_1^{j,T}$ and $\phi_2^{j,T} \in (0, \ 1)$. In addition, $\epsilon_t^{j,P}$ and $\epsilon_t^{j,T}$ are orthogonal *i.i.d.* innovations to $\widehat{\mu_t^{j,P}}$ and $\widehat{\mu_t^{j,T}}$, respectively. The mean and variance of these shocks are given by

$$\begin{bmatrix} \epsilon_t^{j,P} \\ \epsilon_t^{j,T} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{j,P}^2 & 0 \\ 0 & \sigma_{j,T}^2 \end{bmatrix}\right) \ .$$

For total factor productivity growth, it is also assumed that there are permanent and transitory components embedded in the process of z_t:

$$z_t = z_t^P + z_t^T \ ; \tag{3.4}$$

$$z_t^P = \phi^{z,P} z_{t-1}^P + \epsilon_t^{z,P} \ ; \tag{3.5}$$

$$z_t^T = \phi_1^{z,T} z_{t-1}^T + \phi_2^{z,T} z_{t-2}^T + \epsilon_t^{z,T} \ ; \tag{3.6}$$

$$\begin{bmatrix} \epsilon_t^{z,P} \\ \epsilon_t^{z,T} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{z,P}^2 & 0 \\ 0 & \sigma_{z,T}^2 \end{bmatrix} \right) .$$

Once again, $\phi_1^{z,T}$ and $\phi_2^{z,T}$ are auto-regression coefficients that lie within [0, 1]; $\phi^{z,P}$ is close to, but strictly less than one; and $\epsilon_t^{z,P}$ and $\epsilon_t^{z,T}$ are *i.i.d.* normal shocks.

For the rest of the exogenous variables, we simply assume that they follow stationary $AR(2)$ processes, and no permanent components are embedded:

$$g_t = \phi_1^g g_{t-1} + \phi_2^g g_{t-2}^g + \epsilon_t^g \ , \tag{3.7}$$

$$z_t^I = \phi_1^{z^I} z_{t-1}^I + \phi_2^{z^I} z_{t-2}^I + \epsilon_t^{z^I} \ , \tag{3.8}$$

$$\xi_t = \phi_1^{\xi} \xi_{t-1} + \phi_2^{\xi} \xi_{t-2} + \epsilon_t^{\xi} \ , \tag{3.9}$$

where $g_t \equiv log(\widetilde{G_t}/\widetilde{G})$ in equation (3.7) refers to the percentage deviation of the growth rate of government expenditures from its steady state; ϕ_1^s and ϕ_2^s, for $s = \{g,\ z^I,\ \xi\}$, denote auto-regression coefficients; ϵ_t^s is *i.i.d.* normal innovations to variable s_t with mean zero and standard deviation σ_s.

Note that the structure of exogenous processes of z_t, g_t, ξ_t and z_t^I stays fixed across all four model specifications. In the alternative specifications, however, the modeling assumption varies in the permanent components of $\widehat{\mu_t^x}$ and $\widehat{\mu_t^a}$. In specification 2, we eliminate the permanent component of $\widehat{\mu_t^x}$ from its process, and simply assume that the dynamics of $\widehat{\mu_t^x}$ is only captured by the transitory component. Formally, keeping all the other exogenous structure fixed, $\widehat{\mu_t^x}$ takes the following form:

$$\widehat{\mu_t^x} \equiv \widehat{\mu_t^{x,T}} = \phi_1^{x,T} \widehat{\mu_{t-1}^{x,T}} + \phi_2^{x,T} \widehat{\mu_{t-2}^{x,T}} + \epsilon_t^{x,T} \ . \tag{3.10}$$

In specification 3, the permanent component of $\widehat{\mu_t^a}$ is excluded in an analogous fashion. And in the last model specification, neither $\widehat{\mu_t^x}$ nor $\widehat{\mu_t^a}$ is assumed to exhibit persistent long-run growth. Modeling assumption for each specification is summarized in Table 1.

Table 1: Structure of Exogenous Processes: Baseline and Three Alternatives

	z_t		$\widehat{\mu_t^x}$		$\widehat{\mu_t^a}$	
	Permanent	Transitory	Permanent	Transitory	Permanent	Transitory
Baseline	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Specification 2	$\surd$	$\surd$	$-$	$\surd$	$\surd$	$\surd$
Specification 3	$\surd$	$\surd$	$\surd$	$\surd$	$-$	$\surd$
Specification 4	$\surd$	$\surd$	$-$	$\surd$	$-$	$\surd$

Note: "$\surd$" indicates that the variable consists of that component; and "$-$" indicates that the variable does not contain that component.

4 Bayesian Estimation

4.1 Data and Estimation Procedure

In this study, we apply Bayesian method to the state-space representation of the linearized equilibrium conditions and estimate the non-calibrated structural parameters for all model specifications. In particular, we exploit the Markov Chain Monte Carlo (MCMC) approach with a Random Walk Metroplis-Hastings (RW-MH) algorithm to improve computational efficiency. Our RW-MH sampling algorithm generates 100,000 draws from the proposed distribution and has a 50,000-draw burn-in period. The procedure of our Bayesian estimation is standard in the literature.

To estimate the unknown deep parameters, we use quarterly U.S. data ranging from 1949:Q1 to 2006:Q4. The vector of observables Ω_t is listed as follows:

$$\Omega_t = \left[\Delta log\left(Y_t\right),\ \Delta log\left(A_t I_t\right),\ log(C_t/Y_t),\ log(G_t/Y_t),\ log(V_t/Y_t),\ H_t,\ R_t^{rf}\right]' , \tag{4.1}$$

where $\Delta log\left(Y_t\right)$ refers to the growth rate of real per capita GDP; $\Delta log\left(A_t I_t\right)$ denotes the growth rate of real per capita investment; (C_t/Y_t), (G_t/Y_t) and (V_t/Y_t) denote the shares of real per capita consumption, real per capita government expenditures and real per capita total market values in real per capita output, respectively; H_t refers to hours worked; and R_t^{rf} is the real risk-free rate. Detailed data description is provided in Appendix C.

4.2 Calibration

In this study, we calibrate a small set of structural parameters that regulate the dynamics of the aggregate variables in the modeled economy. First, as summarized in Table 2, the representative household's subjective discount factor β is calibrated at 0.99; the steady state share of capital stock in output α is set at 0.3; the steady state depreciation rate and capital utilization rate are calibrated at 0.025 and 1, respectively; and the steady state government-output ratio is set equal to 0.2. Note that these calibration choices are widely used in the real business cycle literature.

Second, following Schmitt-Grohé and Uribe (2012), we set the steady state growth rates of labor-augmenting technology and investment-specific technology at 1.00165 and 0.9957, respectively. Given the calibrated values of μ^x, μ^a and α, the implied steady state values of μ^y and μ^k are 1.0045 and 1.0033, respectively.

In addition, the steady state values of the exogenous variables (namely $\widehat{\mu_t^{x,P}}$, $\widehat{\mu_t^{x,T}}$, $\widehat{\mu_t^{a,P}}$, $\widehat{\mu_t^{a,T}}$, z_t^P, z_t^T, g_t, z_t^I and ξ_t) are all zero. So as to capture the persistent changes in the permanent components (yet without introducing non-stationarity to the equation system), the auto-regression coefficients of $\phi^{x,P}$, $\phi^{a,P}$ and $\phi^{z,P}$ are calibrated at 0.99. Calibration is identical for all model specifications.

Table 2: Calibration

Parameter	Value	Description
β	0.99	Subjective discount factor;
α	0.3	Steady state share of capital;
δ_0	0.025	Steady state depreciation rate;
u_{ss}	1	Steady state value of capital utilization rate;
μ^x	1.00265	Steady state growth rate of labor-augmenting technology;
μ^a	0.9957	Steady state growth rate of investment-specific technology;
G_{ss}/Y_{ss}	0.2	Steady state share of government expenditure in output;
$\phi^{z,P}$	0.99	Auto-regression coefficient of the permanent component z_t^P;
$\phi^{x,P}$	0.99	Auto-regression coefficient of the permanent component $\widehat{\mu_t^{x,P}}$;
$\phi^{a,P}$	0.99	Auto-regression coefficient of the permanent component $\widehat{\mu_t^{a,P}}$.

4.3 Parameter Estimates

Exploiting the RW-MH sampling algorithm, we perform Bayesian estimation of the deep parameters for all model specifications. In the baseline model, the set of the estimated parameter, Θ, is given by

$$\Theta = \begin{pmatrix} \gamma,\ \kappa,\ b_c,\ b_l,\ \chi,\ \delta_2\ \rho^{xg} \\ \phi_1^{x,T},\ \phi_2^{x,T},\ \phi_1^{a,T},\ \phi_2^{a,T},,\ \phi_1^{z,T},\ \phi_2^{z,T},\ \phi_1^{g},\ \phi_2^{g},\ \phi_1^{z^I},\ \phi_2^{z^I},\ \phi_1^{\xi},\ \phi_2^{\xi} \\ \sigma_{x,P},\ \sigma_{x,T},\ \ \sigma_{a,P},\ \sigma_{a,T},\ \sigma_{z,P},\ \sigma_{z,T},\ \sigma_g,\ \sigma_{z^I},\ \sigma_\xi \end{pmatrix}.$$

The first row of Θ consists of the non-calibrated economic parameters; the second row includes the auto-regression coefficients governing the transitional dynamics of the exogenous variables; and the third row incorporates the standard deviation of the corresponding economic shocks.

Table 3 displays the prior distribution of the estimated parameters and reports the statistics characterizing their posterior distribution under the baseline model.[4] In general, we employ flat priors so that the posterior is primarily determined by the likelihood of the data. In the baseline model, the mean estimate of the IES parameter γ is 1.7483, which is consistent with the findings of a large body of the real business cycle literature. Second, the posterior mean of b_c and b_l are 0.7377 and 0.8931, respectively. These estimates imply relatively high degree of habit formation in consumption and leisure. And the posterior mean of δ_2, the parameter that governs the convexity of the depreciation rate function, is 0.1113, which further implies that the elasticity of capital utilization to the rental rate of capital is approximately 0.6.

[4] Parameter estimates for alternative model spcifications are reported in Table D.1 - D.3 in Appendix D. Across all model specifications, the prior distribution of the structural parameters is kept almost identical. The prior distribution in specification 4 is specified with minor difference to avoid the non-positivity of the Hessian matrix arising from estimating the posterior mode.

Table 3: Bayesian Estimation of Structural Parameters : Baseline Model

	Prior			Posterior				
Parameter	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	1.7483	1.7556	0.0375	1.6952	1.8000
κ	Gamma	4	2	3.7927	3.7974	0.0465	3.7496	3.8325
b_c	Beta	0.5	0.2	0.5679	0.5676	0.0021	0.5633	0.5732
b_l	Beta	0.5	0.2	0.7211	0.7210	0.0036	0.7201	0.7224
χ	Gamma	4	2	3.2202	3.2163	0.0280	3.1612	3.2887
δ_2	Gamma	0.1	0.05	0.1113	0.1112	0.0005	0.1110	0.1117
$\phi_1^{x,T}$	Beta	0.6	0.3	0.4047	0.4050	0.0036	0.4014	0.4074
$\phi_2^{x,T}$	Beta	0.2	0.1	0.2085	0.2085	0.0010	0.2080	0.2089
$\phi_1^{a,T}$	Beta	0.6	0.3	0.5439	0.5438	0.0019	0.5407	0.5470
$\phi_2^{a,T}$	Beta	0.2	0.1	0.1417	0.1415	0.0011	0.1409	0.1426
$\phi_1^{z,T}$	Beta	0.6	0.3	0.7731	0.7728	0.0134	0.7716	0.7748
$\phi_2^{z,T}$	Beta	0.2	0.1	0.2268	0.2270	0.0026	0.2251	0.2283
ϕ_1^{ξ}	Beta	0.6	0.3	0.8420	0.8420	0.0060	0.8406	0.8435
ϕ_2^{ξ}	Beta	0.2	0.1	0.1562	0.1563	0.0033	0.1551	0.1572
$\phi_1^{z^I}$	Beta	0.6	0.3	0.5767	0.5763	0.0058	0.5736	0.5807
$\phi_2^{z^I}$	Beta	0.2	0.1	0.3031	0.3031	0.0033	0.3023	0.3037
ϕ_1^{g}	Beta	0.6	0.3	0.7025	0.7029	0.0047	0.6958	0.7099
ϕ_2^{g}	Beta	0.2	0.1	0.1948	0.1948	0.0009	0.1933	0.1963
ρ^{xg}	Beta	0.7	0.3	0.3575	0.3575	0.0141	0.3556	0.3592
$\sigma_{x,P}$	Inverse-Gamma	0.02	Inf.	0.0033	0.0033	0.0004	0.0028	0.0037
$\sigma_{x,T}$	Inverse-Gamma	0.1	Inf.	0.0253	0.0253	0.0033	0.0242	0.0264
$\sigma_{a,P}$	Inverse-Gamma	0.02	Inf.	0.0079	0.0079	0.0006	0.0074	0.0085
$\sigma_{a,T}$	Inverse-Gamma	0.1	Inf.	0.0704	0.0713	0.0024	0.0645	0.0738
$\sigma_{z,P}$	Inverse-Gamma	0.02	Inf.	0.0168	0.0169	0.0028	0.0155	0.0181
$\sigma_{z,T}$	Inverse-Gamma	0.1	Inf.	0.0119	0.0119	0.0009	0.0118	0.0122
σ_{ξ}	Inverse-Gamma	0.1	Inf.	0.0641	0.0640	0.0054	0.0620	0.0665
σ_{z^I}	Inverse-Gamma	0.1	Inf.	0.1394	0.1396	0.0074	0.1316	0.1468
σ_g	Inverse-Gamma	0.1	Inf.	0.0294	0.0294	0.1058	0.0277	0.0312

Overall, our estimates of the deep economic parameters under the baseline model are consistent with findings reported in the real business cycle literature. Compared with Schmitt-Grohé and Uribe (2012) and Avdjiev (2009) whose empirical practice is based on similar theoretical framework, our estimation achieves non-trivial improvements primarily along two dimensions.[5] First, Avdjiev (2009) finds substantially lower degree of habit formation in consumption and leisure, which is inconsistent with previous findings. Second, the posterior mean estimates of the Frisch elasticity parameter (χ) and the parameter governing the investment adjustment cost (κ) are 3.2202 and 3.7927, respectively. These values are closer to the generally agreed values in the real business cycle literature than those reported in Schmitt-Grohé and Uribe (2012).

As displayed in Table 6 - 8 in Appendix D, estimates of the economic parameters are robust to alternative specifications on the exogenous processes. Noticeable differences are discussed as follows. First, the posterior mean of γ under specifications 2 and 4 is around 1.35, which is slightly lower than its baseline counterpart, and the mean estimate under specification 3 is 2.2855, which is marginally higher than 2, the widely accepted upper bound of the IES parameter. Across all model specifications, however, the IES estimates are greater than 1, indicating that, upon the arrival of macroeconomic shocks, the income effect dominates the substitution effect. Second, eliminating the persistent long-run components in labor-augmenting technology and investment-specific technology, specification 4 yields lower estimates of the habit formation parameters than the alternatives. And the posterior mean of δ_2 is only one-quarter as large as those under the other alternative models. Finally, while not contradicting the empirical evidence borrowed from the real business cycle literature, the posterior estimates of χ and κ across four model specifications do not exhibit any consistent pattern.

For the parameters regulating the stochastic processes of the exogenous variables, first, the posterior mean estimates of $\sigma_{x,T}$, $\sigma_{z,P}$, $\sigma_{z,T}$, σ_{z^I} and σ_g are almost identical across all model specifications. Second, specification 3 yields the mean estimate of σ_ξ twice as large as those under other alternatives. In addition, under specification 2 and 3, the posterior mean of $\sigma_{a,T}$ is around 0.03, which is twice as large as that under specification 4, and in the meantime even less than one half of its baseline counterpart. For other volatility parameters, namely $\sigma_{x,P}$ and $\sigma_{a,P}$, it is found that: (1) under specification 2, where the permanent component of labor-augmenting technology is eliminated, the estimated volatility of shocks to long-run investment-specific technology growth ($\sigma_{a,P}$) is twice as much as the one under the baseline model; and (2) under specification 3, where the long-run component of investment-specific technology is excluded, the estimated volatility of shocks to long-run labor-augmenting technology growth ($\sigma_{x,P}$) is also twice as large as its baseline counterpart.

[5] A major difference between our model and those of Schmitt-Grohé and Uribe (2012) and Avdjiev (2009) is that we do not consider anticipated shocks.

5 Variance Decomposition and Model Comparison

5.1 Unconditional Variance Decomposition

So as to identify the major sources of fluctuations in aggregate variables and asset prices, in this subsection, we perform unconditional variance decomposition and attempt to quantify the share of the predicted error variance of the seven observables traceable to each of the macroeconomic shocks. Decomposition statistics under the baseline model are reported in Table 4, and findings under the alternative models are presented in Appendix D.

First, unconditional variance decomposition highlights the importance of economic shocks to the permanent components of technology growth to accounting for the business cycles. The innovations to the permanent components of total factor productivity growth, labor-augmenting technology growth, and investment-specific technology growth jointly explain 35% - 40% of the predicted error variance of output growth, investment growth, the share of consumption in total output, and hours. While merely 16% of the variation in government spending to output ratio is due to $\epsilon_t^{x,P}$, $\epsilon_t^{a,P}$ and $\epsilon_t^{z,P}$, these shocks explains 49% of the predicted error variance of the risk-free rate. In addition, it is more or less surprising to see that the shocks to persistent long-run technology growth jointly account for almost 86% of the predicted error variance of the share of total market values in output, whereas the contribution of innovations to the transitory components of technology growth is only 14%.

Second, the decomposition statistics sheds light on the identification of the key driving forces of the business cycles, and indicates that previous studies might have underestimated the importance of investment-related technology shocks. As shown in Table 4, shocks to the permanent and the transitory components of the investment-specific technology growth explains the largest fraction of fluctuations in output growth, investment growth and the risk-free rate. Shocks to the permanent component of the investment-specific technology growth, $\epsilon_t^{a,P}$, is also the second important predictor of variation in consumption-output ratio. While the most important contributor to variation in $log(C_t/Y_t)$ is the shocks to the transitory component of total factor productivity growth $\epsilon_t^{z,T}$, its quantified contribution is merely marginally higher than that of $\epsilon_t^{a,P}$. However, $\epsilon_t^{z,T}$ is indeed one of the most important factor explaining the predicted error variance of consumption to output ratio and government spending to output ratio. For the asset market variable, it is once again surprising to find that nearly 88% of the variation in the ratio of total market values to output is accounted for by innovations to the permanent and the transitory components of investment-specific technology growth. Except for shocks to the transitory component of labor-augmenting technology growth, other macroeconomic innovations play literally no role in explaining asset price movements.

Compared with the evidence from the existing literature, our unconditional variance decomposition is consistent with the findings of Justiniano, Primiceri and Tambalotti (2008), who argue that shocks to investment-specific technology play a central role in driving aggregate fluctuations. However, these results are in sharp contrast to those of Avdjiev (2009) and Schmitt-Grohé and Uribe (2012). The apparent discrepancy might arise from different assumptions made on the underlying modeled economy (such as preference specifications, in-

Table 4: Unconditional Variance Decomposition : Baseline

	Shock	g^Y	g^{AI}	$log\left(\frac{C}{Y}\right)$	$log\left(\frac{G}{Y}\right)$	$log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	0.1533	0.1046	0.0292	0.0339	0.1102	0.0295	0.1442
	$\epsilon^{a,P}$	0.2441	0.2869	0.3327	0.1123	0.7501	0.3176	0.2128
	$\epsilon^{z,P}$	0.0070	0.0093	0.0040	0.0150	0.0007	0.0073	0.1326
Sum		0.4044	0.4008	0.3659	0.1612	0.8610	0.3544	0.4896
Transitory	$\epsilon^{x,T}$	0.1450	0.0371	0.0049	0.0050	0.0048	0.0036	0.0350
	$\epsilon^{a,T}$	0.3797	0.3850	0.1294	0.0925	0.1270	0.1021	0.2746
	$\epsilon^{z,T}$	0.0020	0.0028	0.3685	0.5037	0.0020	0.5118	0.1534
	ϵ^{ξ}	0.0054	0.0010	0.1240	0.1940	0.0004	0.0241	0.0050
	ϵ^{z^I}	0.0565	0.1727	0.0072	0.0322	0.0048	0.0039	0.0125
	ϵ^{g}	0.0071	0.0006	0.0002	0.0113	0.0001	0.0001	0.0300
Sum		0.5957	0.5992	0.6342	0.8387	0.1391	0.6456	0.5105

corporation of certain variables and sectors, and so forth), but usually seems not quantitatively traceable.[6] As shown in the rest of this subsection, even small changes in the specification on the stochastic processes of certain exogenous variables, while keeping everything else constant, are likely to lead to remarkably distinct model implications.

Under specification 2, where the persistent component of labor-augmenting technology is omitted, innovations to the persistent long-run growth of total factor productivity and investment-specific technology remain to be the major drivers of aggregate fluctuations. And shocks to the permanent and the transitory components of investment-specific technology growth once again contribute the largest fraction to accounting for the predicted error variance of key aggregate variables. However, the importance of permanent and transitory total factor productivity shocks diminishes in a relatively significant manner. While the contribution of total factor productivity shocks to risk-free rate increases by more than 30%, the share of variation in consumption and in government expenditures explained by $\epsilon_t^{z,P}$ and $\epsilon_t^{z,T}$ declines sharply to nearly 0.5%.

For specification 3, where persistent long-run investment-specific technology growth is excluded, it is found that variation in output growth, investment growth, consumption and total market values are primarily driven by shocks to the permanent and the transitory components of labor-augmenting technology growth. Shocks to government expenditures explain the largest share of the government expenditures to output ratio, and the second largest share of variation in consumption-output ratio. The effect of shocks to other exogenous variables is quantitatively close to zero. Under specification 4, where total factor productivity growth is the only exogenous variable consisting of a permanent component, shocks to total factor productivity growth and to labor-augmenting technology growth explain 48% and 28% of the variation in output growth, respectively; innovations to transitory investment-specific technology growth are the major contributors to fluctuations in investment, consumption and total market values; shocks to z_t^I explain the second largest share of the predicted error variance

[6] For instance, while Avdjiev (2009) employs similar theoretical framework to ours, several important variables, such as government spending and preference shocks, are missing. Schmitt-Grohé and Uribe (2012) employ the Jamovich-Rebelo utility function and take into account both anticipated and unanticipated shocks.

of investment growth; preference shocks explain 78% of the variation in hours; and shocks to government expenditures literally plays no role.

Comparing the baseline variance decomposition with those under the three alternatives unambiguously demonstrates that the identification of the key drivers of aggregate fluctuations hinges critically upon researchers' specification on the structure of the exogenous processes. The relatively high sensitivity of decomposition statistics to small changes in the assumption on the exogenous processes is not a trivial issue, because the true data generating processes of the exogenous variables are usually unobservable, and making arbitrary assumption on their unobserved structure can potentially lead to biased model implications. To minimize the impact of misspecification on model inference, we exploit Bayesian model comparison technique to identify the model specification that is mostly supported by the data. This issue is discussed in section 5.3.

5.2 Conditional Variance Decomposition

In addition to unconditional variance decomposition, we exploit conditional variance decomposition to investigate the key drivers of aggregate fluctuations from a dynamic perspective. First, as displayed in Appendix E, decomposition results under the baseline model indicate that both the long-run and the short-run forecasting error variance of output growth is primarily explained by shocks to the transitory components of labor-augmenting and investment-specific technology growth; the contribution of shocks to the permanent components of these two technology growth rates is quantitatively small at short forecasting horizons, but increases dramatically in the long-run. In particular, for investment-specific technology growth, the contribution of shocks to its transitory component dominates the contribution of those to its permanent component in the short-run, but becomes dominated at long forecasting horizons. For macroeconomic innovations to the technology rate of transforming investment goods into capital, ϵ^{z^I}, its contribution to the variation in output growth, investment growth, and the shares of consumption and government expenditures in output is non-negligible, but diminishes as the forecasting horizon increases. Similar to ϵ^{z^I}, government expenditure shocks induce more than 50% of the variation in government expenditures to output ratio in the short-run, but its contribution falls sharply at longer forecasting horizons and eventually becomes zero.

When the permanent component of investment-specific technology growth is omitted, model implication under specification 2 is partly consistent with its baseline counterpart. Noticeable distinction, however, lies in the fact that a set of exogenous shocks that play trivial roles in the baseline model become significant contributors to inducing short-run aggregate fluctuations. For instance, at short forecasting horizons, shocks to the permanent component of total factor productivity explains a large fraction of the forecasting error variance of output; the contribution of preference shocks to inducing short-run fluctuations in consumption-output ratio is almost identical to that of shocks to the permanent component of investment-specific technology growth; and over 70% of the short-run variation in government expenditures to output ratio can be attributable to government expenditure shocks.

It is worth mentioning that the findings of the conditional variance decomposition under each model specification further confirms that inference about the major driving forces of the business cycles, regardless of the forecasting horizons, hinges critically upon the specification on the structure of the exogenous processes. Presumption on the unobserved stochastic processes can potentially lead to biased model implications. Therefore, evaluating a set of competing alternatives based on the likelihood of the data seems necessary to mitigate the impact of misspecification.

5.3 Bayesian Model Comparison

In appendix F, we plot the series of data on the seven observed variables along with their smoothed counterparts. In general, all model specifications fit the observables reasonably well. Therefore, the statistics of the goodness-of-fit can hardly help to assess the relative plausibility of these model specifications. Hence, we compute the Bayesian odd ratio of each model to figure out which specification is mostly favored by the likelihood of the data. Since it is difficult to verify the existence of persistent long-run growth in labor-augmenting technology and investment-specific technology *a priori,* , equal priors are assigned to the baseline model and the three alternatives. We find that specification 2 and the baseline model yield the highest and the 2nd highest marginal density, respectively; and under specification 3, where the variance decomposition implications are dramatically distinct from other alternatives, has the lowest marginal density. In addition, even though the marginal density of the baseline model is slightly lower than that of specification 2, the odd ratio test overwhelmingly supports the specification that labor-augmenting technology growth does not consist of a permanent component. Given that the odd ratio under specification 2 is surprisingly large, the estimated posterior probability is 1. Therefore, these findings indicate that previous studies might have overlooked persistent long-run investment-specific technology growth, and thus fail to take into account its effect on driving the business cycles.

Table 5: Bayesian Model Comparison

	Baseline	Specification 2	Specification 3	Specification 4
Prior Probability	0.25	0.25	0.25	0.25
Log Marginal Density	3512.13	3575.87	3355.44	3396.71
Odd Ratio	1.00	4.84×10^{27}	0.00	0.00
Posterior Probability	0.00	1.00	0.00	0.00

6 Aggregate Dynamics and Asset Price Movements

The remaining task of this study is to investigate how aggregate variables make dynamic adjustments to the macroeconomic innovations. In Appendix G, Figure G.1 displays the impulse response functions of the investigated variables to a one standard deviation (S.D. hereafter) negative shock to the permanent component of investment-specific technology growth under specification 2. It is worth mentioning that a shock of positive value to investment-specific technology growth is interpreted as a negative investment-specific technology shock. This is because, in a decentralized market, the variable A_t measures the relative price of investment goods, and hence, an increase in A_t implies lower efficiency of investment-specific technology. When a one S.D. negative shock of $\epsilon_t^{a,P}$ is imposed, investment initially increases, but starts to decrease after 10 quarters, and eventually becomes negative at long forecasting horizons. While seemingly perplexing, the fact that investment does not fall immediately in response to a negative investment-productivity shock is not difficult to understand. It is because, when the negative shock of $\epsilon_t^{a,P}$ becomes realized, forward-looking economic agent is aware that the shock would induce permanent changes in investment-specific technology growth. Given its persistent nature, the representative household expects that the relative price of investment would continue to rise in the future. As a consequence, the agent would tend to revise her investment plan by investing more immediately (when the price of investment is relatively cheap) and investing less in the future (when the price of investment is relatively expensive).

Second, Figure G.1 suggests that consumption and leisure increase in response to $\epsilon_t^{a,P}$. Given that negative investment-specific technology shocks induce higher relative price of investment goods with respect to consumption goods, the representative household would tend to substitute away from investment and simply choose to consume more. In addition, once exposed to a negative investment-specific technology shock, the representative firm also has incentives to re-optimize its production plan. Figure G.1 shows that the utilization rate of capital falls as the relative price of investment increases. The magnitude of such adjustment in capital utilization is large enough to offset the effect of reduction in investment, and consequently leads to an increase in effective capital stock. Since higher level of effective capital stock reduces the marginal productivity of capital, the relative price of capital to labor also decreases, resulting in less demand for labor. Therefore, leisure declines in response to a negative investment-specific technology shock, the impact of which can potentially be large enough to induce decreases in output at business cycle frequencies. For asset price movements, it is somehow surprising to find that a negative investment-specific technology shock increases total market values.[7] A potential explanation, however, could be that, even though output falls and risk free rate rises in response to a negative investment-productivity shock, investment decreases by more, which generates a sufficiently large dividend effect such that the expected sum of future dividend payment increases enough to offset the risk-free rate effect and eventually leads to an increase in total market values.

For a one S.D. $\epsilon_t^{a,T}$ shock, Figure G.2 indicates that the pattern of the impulse responses of the investigated aggregate variables is largely similar to that in the case where $\epsilon_t^{a,P}$ is imposed. The similarity is unambiguously due to the fact both $\epsilon_t^{a,P}$ and $\epsilon_t^{a,T}$ affect the economy through the same channel (decreasing the growth rate of investment-specific technology). Nevertheless,

[7]Admittedly, our theoretical framework, as well as other standard business cycle models, might fail to provide fully adequate explanations to fluctuations in asset prices. Because, instead of explicitly modeingl the stochastic discount factor, these types of analytical framework usually assume that asset price movements are purely driven bythe risk-free rate and future cash-flows.

we observe that the dynamic responses of all macroeconomic variables induced by $\epsilon_t^{a,T}$ are not only less persistent but also in much smaller magnitude. These findings are consistent with the definition of these two types of shocks.

Under specification 2, the other exogenous variable assumed to consist of a permanent component is the total factor productivity growth. Figure G.3 shows that, the arrival of a one S.D. $\epsilon_t^{z,P}$ shock has an immediate positive effect on investment, but such impact diminishes gradually at longer forecasting horizons. Capital utilization rate falls by 3% instantaneously, and returns to its original level in a few quarters afterward. And a higher level of investment, combined with declined utilization rate, increases capital stock. In addition, Figure G.3 suggests that a positive total factor productivity shock tend to induce persistent increases in consumption and leisure, whereas its effect on the risk-free rate is not only negative but also short-lived. In terms of production, a positive $\epsilon_t^{z,P}$ shock leads to persistent increases in total output over time. Conditional on higher levels of output and investment, our model predicts that their net effect on future dividend payment is negative. As a consequence, the interest rate effect slightly dominates the dividend effect, and thus, induces extremely short-lived increases in total market values. Compared with $\epsilon_t^{z,P}$, a one S.D. $\epsilon_t^{z,T}$ shock induces similar pattern of dynamic adjustments of the macroeconomic variables, but its effect is not only short-lived but also smaller in magnitude. In addition, it is worth pointing out that the magnitude of changes in macroeconomic variables induced by total factor productivity shocks is merely one-tenth as large as those generated by the investment-specific technology shocks, which further confirms that investment-specific technology shock is very likely to be the major driving force of the business cycles.

For a one S.D. shock to the transitory component of labor-augmenting technology growth, it induces instantaneous decreases in investment and the capital utilization rate. But its negative impact on these two variables becomes positive after 5 quarters, and eventually dies out at longer forecasting horizons. The net effect of investment and capital utilization rate responses on capital stock is initially negative and diminishes as the forecasting horizon increases. Upon the arrival of $\epsilon_t^{z,T}$, our model predicts that the representative household reduce her consumption by fairly small amount. While hours increase in response to a positive labor-augmenting technology shock, short-run effective capital stock does decrease by a sufficiently large amount so that total output falls in the first 10 quarters. When effective capital stock resumes to its original level and hours reach its peak in the 10th quarter, changes in output become positive. However, the positive effect of labor-augmenting technology shock on output is not long-lasting and eventually dies out at longer forecasting horizons. In addition, it is found that positive labor-augmenting technology shock reduces dividends and results in a harp-shaped dynamic path. In the meanwhile, interest rate falls by 1.5% immediately after the shock is imposed, and goes back to its original level in 10 periods. Combined with the dynamic response of total market values, these pieces of evidence indicate that positive labor-augmenting technology shocks tend to increase total market values in the short-run through the interest rate channel, and reduce total market values in the long-run via the dividend channel.

As suggested in the literature, another potentially important source of aggregate fluctuations is the shock to the marginal efficiency of investment. Under our framework, investment, capital utilization and hours all respond positively upon the arrival of a positive one S.D. ϵ^{z^I} shock, and gradually fall below their original levels after 15 forecasting periods. In the long-run, ϵ^{z^I} produces negative impact on investment, capital utilization rate and hours. However, the magnitude of its long-run negative effect is smaller than that of its initial positive effect. In addition, capital stock responds positively to a positive ϵ^{z^I} shock throughout the forecasting horizons, and the dynamics of total output is similar to those of investment and hours. It is also found that the dynamic path of firm's profit is close to the mirror image of that of investment (and output). As risk-free rate only responds positively to the shock to z^I during the initial periods, impulse responses of total market values almost replicate the dynamic path of dividend payment.

For other macroeconomic innovations, first, we find that, while relatively small in magnitude, a one S.D. preference shock is able to induce permanent changes in consumption, labor supply, investment, output and total market values. Second, the effect of government spending shocks on the aggregate variables is instantaneous, but not long-lasting. Finally, it is worth pointing out that several perplexing phenomena are identified in the impulse responses analysis using other model specifications. For instance, under the baseline model, shocks to the transitory component of total factor productivity seem to induce permanent changes in a subset of the aggregate variables; and total market values fall throughout the entire forecasting periods responding to positive shocks to the permanent and the transitory components of labor-augmenting technology.[8]

7 Conclusion

In this study, we develop a neoclassical growth model and seek to investigate the major driving forces of the business cycles. So as not to rule out the possibility that the growth rates of total factor productivity, labor-augmenting technology and investment-specific technology consist of persistent long-run components, we specify a baseline model along with three competing alternatives through varying the assumption on the structures of the stochastic processes of these exogenous variables, and exploit Bayesian inference methods to assess their relative plausibility.

Quantitative analysis based on variance decomposition, Bayesian odd ratio test and impulse response functions indicates that the identification of the key drivers of the aggregate fluctuations is heavily contingent upon researchers' assumption on the structure of the exogenous processes. In particular, empirical evidence suggests that previous studies might have overlooked the persistence embedded in the process of investment-specific technology growth, and thus underestimate its importance to driving the business cycles. According to our findings, shocks to investment-specific technology account for a vast majority of the short-run and the long-run predicted error variance of output growth, investment growth, the

[8]These anomalous findings are not reported in the paper. However, they are available upon request.

share of government expenditures in output, hours, and total market values to output ratio; preference shocks explain over 50% of the short-run fluctuations in the consumption-output ratio; a large fraction of the short-run fluctuations in hours is attributable to shocks to the permanent component of labor-augmenting technology; and government spending shocks are only important to explaining the movements in the government expenditures to output ratio in short forecasting horizons.

Compared with previous empirical studies which employ similar theoretical framework, parameter estimation in this paper achieves noticeable improvements along a few dimensions. However, this study has several limitations needed to be overcome in future work. First, considering that it would be difficult to infer a large variance-covariance matrix of the exogenous shocks from a relatively small sample with merely seven observables, we simply assume that these macroeconomic innovations are orthogonal to each other. Admittedly, this assumption seem not quite plausible. Relaxing the orthogonality assumption might help to resolve several perplexing phenomena reported in the impulse responses analysis, but needs to be based on more reasonable identification (or estimation) schemes capable of accurately extracting information from the low frequency data. In addition, similar to other neoclassical models using standard non-recursive preference specifications, the theoretical framework adopted in this paper seems not able to adequately capture the movements in asset prices. Modern asset pricing literature argues that asset price movements are primarily driven by variation in the stochastic discount factor. The long-run risks literature, which achieves remarkable improvements on replicating the asset pricing facts in partial equilibrium framework, highlight the importance of long-run consumption growth and its conditional volatility to resolving several asset pricing puzzles. In contrast, macroeconomic innovations in our modeled economy can only affect asset prices through the interest rate channel and the dividend channel. To adequately capture the interactions between asset prices and the market fundamentals, it seems necessary to incorporate recursive (e.g. Epstein-Zin) preference specifications and properly model persistent long-run consumption growth in general equilibrium models.

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Appendices

A Equilibrium Conditions in Stationary Form

In Appendix A, we transform the equilibrium conditions described in section 2 into their corresponding stationary forms. First, we define the stochastic trend of output as $X_t^Y = A_t^{\alpha/(1-\alpha)} X_t$, and the stochastic trend of capital stock as $X_t^K = A_t^{1/(1-\alpha)} X_t$. Then, we detrend the non-stationary variables as follows:

$$\widetilde{Y_t} = \tfrac{Y_t}{X_t^Y};\ \widetilde{K_t} = \tfrac{K_t}{X_{t-1}^K};\ \widetilde{C_t} = \tfrac{C_t}{X_t^Y};\ \widetilde{I_t} = \tfrac{I_t}{X_t^K};\ \widetilde{G_t} = \tfrac{G_t}{X_t^G};$$

$$\widetilde{\Lambda_t} = \tfrac{\Lambda_t}{\left(X_t^Y\right)^{-\gamma}};\ \widetilde{Q_t} = \tfrac{Q_t}{A_t};\ \widetilde{P_t} = \tfrac{P_t}{X_t^Y};\ \widetilde{V_t} = \tfrac{V_t}{X_t^Y}\ .$$

Note that the growth rates of output and capital are given by

$$\mu_t^y = \frac{X_t^Y}{X_{t-1}^Y} = \frac{X_t}{X_{t-1}} \left(\frac{A_t}{A_{t-1}}\right)^{\alpha/(\alpha-1)} = \mu_t^x \left(\mu_t^a\right)^{\alpha/(\alpha-1)}\ ; \tag{A.1}$$

$$\mu_t^k = \frac{X_t^K}{X_{t-1}^K} = \frac{X_t}{X_{t-1}} \left(\frac{A_t}{A_{t-1}}\right)^{1/(\alpha-1)} = \mu_t^x \left(\mu_t^a\right)^{1/(\alpha-1)}\ . \tag{A.2}$$

In addition, given our assumption that

$$X_t^G = \left(X_{t-1}^G\right)^{\rho_{xg}} \left(X_{t-1}^Y\right)^{1-\rho_{xg}}\ ,$$

we define

$$X_t^g = \frac{X_t^G}{X_t^Y}\ .$$

Therefore, we can rewrite the equilibrium conditions in the following stationary form:

$$\widetilde{K_{t+1}} = \left[1 - \delta\left(u_t\right)\right] \frac{\widetilde{K_t}}{\mu_t^k} + e^{z_t^I} \widetilde{I_t} \left[1 - \Phi\left(\frac{\widetilde{I_t}\mu_t^k}{\widetilde{I_{t-1}}}\right)\right]\ , \tag{A.3}$$

$$\widetilde{C_t} + \widetilde{I_t} + \widetilde{G_t} x_t^g = \widetilde{Y_t}\ . \tag{A.4}$$

$$\widetilde{Y_t} = e^{z_t} \left(\frac{u_t \widetilde{K_t}}{\mu_t^k}\right)^{\alpha} \left(H_t\right)^{1-\alpha}\ , \tag{A.5}$$

$$\begin{aligned} \widetilde{\Lambda_t} \quad = \quad & e^{\xi_t}\left(\widetilde{C_t} - b_c \tfrac{\widetilde{C_{t-1}}}{\mu_t^y}\right)^{-\gamma} \left(\tau + b_l H_{t-1} - H_t\right)^{\chi(1-\gamma)} \\ & -\beta b_c E_t \left\{ e^{\xi_{t+1}} \left(\widetilde{C_{t+1}}\mu_{t+1}^y - b_c \widetilde{C_t}\right)^{-\gamma} \left(\tau + b_l H_t - H_{t+1}\right)^{\chi(1-\gamma)} \right\}\ ; \end{aligned} \tag{A.6}$$

$$\widetilde{\Lambda_t} e^{z_t} (1-\alpha) \left(\frac{u_t \widetilde{K_t}}{\mu_t^k}\right)^{\alpha} (H_t)^{-\alpha} = \chi e^{\xi_t} \left(\widetilde{C_t} - b_c \frac{\widetilde{C_{t-1}}}{\mu_t^y}\right)^{1-\gamma} (\tau + b_l H_{t-1} - H_t)^{\chi(1-\gamma)-1}$$

$$-\chi\beta b_l E_t \left\{ e^{\xi_{t+1}} \left(\widetilde{C_{t+1}} \mu_{t+1}^y - b_c \widetilde{C_t}\right)^{1-\gamma} (\tau + b_l H_t - H_{t+1})^{\chi(1-\gamma)-1} \right\} ; \tag{A.7}$$

$$\widetilde{\Lambda_t} = \widetilde{\Lambda_t}\widetilde{Q_t} e^{z_t^I} \left[1 - \Phi\left(\frac{\widetilde{I_t}\mu_t^k}{\widetilde{I_{t-1}}}\right) - \left(\frac{\widetilde{I_t}\mu_t^k}{\widetilde{I_{t-1}}}\right) \Phi'\left(\frac{\widetilde{I_t}\mu_t^k}{\widetilde{I_{t-1}}}\right)\right] \tag{A.8}$$

$$+\beta E_t \left\{ \widetilde{\Lambda_{t+1}} \widetilde{Q_{t+1}} \mu_{t+1}^a \left(\mu_{t+1}^y\right)^{-\gamma} e^{z_{t+1}^I} \left(\frac{\widetilde{I_{t+1}\mu_{t+1}^k}}{\widetilde{I_t}}\right)^2 \Phi'\left(\frac{\widetilde{I_{t+1}\mu_{t+1}^k}}{\widetilde{I_t}}\right) \right\} ;$$

$$\widetilde{\Lambda_t}\widetilde{Q_t} = \beta E_t \left\{ \widetilde{\Lambda_{t+1}} \mu_{t+1}^a \left(\mu_{t+1}^y\right)^{-\gamma} \left[e^{z_{t+1}} \alpha u_{t+1}^{\alpha} \left(\frac{\widetilde{K_{t+1}}}{\mu_{t+1}^k}\right)^{\alpha-1} (H_{t+1})^{1-\alpha} + \widetilde{Q_{t+1}} \left(1 - \delta\left(u_{t+1}\right)\right) \right] \right\} ; \tag{A.9}$$

$$e^{z_t} \alpha u_t \left(\frac{u_t \widetilde{K_t}}{\mu_t^k}\right)^{\alpha-1} (H_t)^{1-\alpha} = \widetilde{Q_t} \delta'(u_t) ; \tag{A.10}$$

$$\widetilde{P_t} = \alpha \widetilde{Y_t} - \widetilde{I_t} ; \tag{A.11}$$

$$R_t^{rf} = \frac{1}{\beta} E_t \left[\frac{\widetilde{\Lambda_t}}{\widetilde{\Lambda_{t+1}}} \left(\mu_{t+1}^y\right)^{\gamma} \right] ; \tag{A.12}$$

$$\widetilde{V_t} = \beta E_t \left[\left(\frac{\widetilde{\Lambda_{t+1}}}{\widetilde{\Lambda_t}}\right) \left(\mu_{t+1}^y\right)^{1-\gamma} \left(\widetilde{V_{t+1}} + \alpha \widetilde{Y_{t+2}} - \widetilde{I_{t+2}}\right) \right] ; \tag{A.13}$$

$$x_t^g = \left(x_{t-1}^g\right)^{\rho_{xg}} \left(\mu_t^y\right)^{-1} . \tag{A.14}$$

Therefore, the complete set of equilibrium conditions in stationary form are given by equation (A.1) - (A.14).

B Analytical Solution to the Steady State Equilibrium

Throughout Appendix B, for any variable J_t, we drop the time index t and let J_{ss} denote its steady state. Given the parameters and the calibrated values of a subset of the modeled variables, it is straightforward to show that $\mu^k_{ss} = \mu^i_i$. Therefore, equation (A.6) evaluated at the steady state implies that

$$\widetilde{Q_{ss}} = 1\ . \tag{B.1}$$

Also, from equation (A.10), we observe that

$$\frac{\widetilde{K_{ss}}}{\mu^k_{ss}} = \left(\frac{\delta_1}{\alpha}\right)^{1/(\alpha-1)} H_{ss}\ . \tag{B.2}$$

Plugging equation (B.1) and (B.2) into (A.9), we have

$$\left(\frac{1}{\beta}\right)\frac{\left(\mu^y_{ss}\right)\gamma}{\mu^a_{ss}} = \delta_1 + 1 - \delta_0\ . \tag{B.3}$$

So as to ensure that the steady state value of capital utilization rate, u_{ss}, is unity, δ_1 is implicitly determined by equation (A.8) - (A.10).

Rewrite equation (B.2) as

$$\frac{\widetilde{K_{ss}}}{\mu^k_{ss}}\left(H_{ss}\right)^{-1} = \left(\frac{\delta_1}{\alpha}\right)^{1/(\alpha-1)}\ . \tag{B.4}$$

Then, plugging equation (B.4) into (A.7) yields

$$\begin{aligned}\widetilde{\Lambda_{ss}}\left(1-\alpha\right)\left(\tfrac{\delta_1}{\alpha}\right)^{\alpha/(\alpha-1)} \quad = \quad & \chi\left(\widetilde{C_{ss}} - b_c\frac{\widetilde{C_{ss}}}{\mu^y_{ss}}\right)^{1-\gamma}\left(\tau + b_l H_{ss} - H_{ss}\right)^{\chi(1-\gamma)-1} \\ & -\chi\beta\left(\widetilde{C_{ss}}\mu^y_{ss} - b_c\widetilde{C_{ss}}\right)^{1-\gamma}\left(\tau + b_l H_{ss} - H_{ss}\right)^{\chi(1-\gamma)-1}\ .\end{aligned}$$

Let $M = (1-\alpha)\left(\frac{\delta_1}{\alpha}\right)^{\alpha/(\alpha-1)}$, and rewrite the above equation as

$$\begin{aligned}\widetilde{\Lambda_{ss}}M \quad = \quad & \chi\left(\widetilde{C_{ss}} - b_c\frac{\widetilde{C_{ss}}}{\mu^y_{ss}}\right)^{1-\gamma}\left(\tau + b_l H_{ss} - H_{ss}\right)^{\chi(1-\gamma)-1} \\ & -\chi\beta b_l\left(\widetilde{C_{ss}}\mu^y_{ss} - b_c\widetilde{C_{ss}}\right)^{1-\gamma}\left(\tau + b_l H_{ss} - H_{ss}\right)^{\chi(1-\gamma)-1}\ .\end{aligned} \tag{B.5}$$

In the steady state, equation (A.6) suggests that

$$\begin{aligned}\widetilde{\Lambda_{ss}} &= \left(\mu^y_{ss}\right)^{\gamma}\left(\mu^y_{ss}\widetilde{C_{ss}}-b_c\widetilde{C_{ss}}\right)^{-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)} \\ &\quad -\beta b_c\left(\widetilde{C_{ss}}\mu^y_{ss}-b_c\widetilde{C_{ss}}\right)^{-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)}\ .\end{aligned} \tag{B.6}$$

Combining equation (B.5) and (B.6) we have

$$\begin{aligned}&\chi\left(\mu^y_{ss}\right)^{\gamma-1}\left(\mu^y_{ss}\widetilde{C_{ss}}-b_c\widetilde{C_{ss}}\right)^{1-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)-1}-\chi\beta b_l\left(\widetilde{C_{ss}}\mu^y_{ss}-b_c\widetilde{C_{ss}}\right)^{1-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)-1} \\ &=M\left(\mu^y_{ss}\right)^{\gamma}\left(\mu^y_{ss}\widetilde{C_{ss}}-b_c\widetilde{C_{ss}}\right)^{-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)}-\beta b_c\left(\widetilde{C_{ss}}\mu^y_{ss}-b_c\widetilde{C_{ss}}\right)^{-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)}\ .\end{aligned} \tag{B.7}$$

Dividing both sides of equation (B.7) by $\left(\widetilde{C_{ss}}\mu^y_{ss}-b_c\widetilde{C_{ss}}\right)^{-\gamma}\left(\tau+\left(b_l-1\right)H_{ss}\right)^{\chi(1-\gamma)-1}$ yields

$$\chi\left(\mu^y_{ss}-b_c\right)\left[\left(\mu^y_{ss}\right)^{\gamma-1}-\beta b_l\right]\widetilde{C_{ss}}=M\left[\left(\mu^y_{ss}\right)^{\gamma}-\beta b_c\right]\left[\tau+\left(b_l-1\right)H_{ss}\right]\ ,$$

by which we can solve for $\widetilde{C_{ss}}$ using H_{ss}:

$$\widetilde{C_{ss}}=\frac{M\left[\left(\mu^y_{ss}\right)^{\gamma}-\beta b_c\right]\left[\tau+\left(b_l-1\right)H_{ss}\right]}{\chi\left(\mu^y_{ss}-b_c\right)\left[\left(\mu^y_{ss}\right)^{\gamma-1}-\beta b_l\right]}\ . \tag{B.8}$$

Using equation (B.2) and (A.3), we are able to solve for $\widetilde{I_{ss}}$ (in terms of H_{ss}):

$$\widetilde{I_{ss}}=H_{ss}\left(\mu^k_{ss}+\delta_0-1\right)\left(\frac{\delta_1}{\alpha}\right)^{1/(\alpha-1)}\ . \tag{B.9}$$

In addition, equation (A.5) implies that

$$\widetilde{Y_{ss}}=H_{ss}\left(\frac{\delta_1}{\alpha}\right)^{\alpha/(\alpha-1)} \tag{B.10}$$

Since we calibrate the steady state share of government expenditures in output gy, it suggests that

$$gy\widetilde{Y_{ss}}=\widetilde{G_{ss}}x^g_{ss}\ . \tag{B.11}$$

Given the solution to $\widetilde{C_{ss}}$, $\widetilde{I_{ss}}$ and $\widetilde{Y_{ss}}$, plugging equation (B.8) - (B.11) into (A.4) yields

$$\begin{aligned}&\left\{(1-gy)\left(\tfrac{\delta_1}{\alpha}\right)^{\alpha/(\alpha-1)}-\left(\mu^k_{ss}+\delta_0-1\right)\left(\tfrac{\delta_1}{\alpha}\right)^{1/(\alpha-1)}-\tfrac{M\left[\left(\mu^y_{ss}\right)^{\gamma}-\beta b_c\right]\left[\left(b_l-1\right)H_{ss}\right]}{\chi\left(\mu^y_{ss}-b_c\right)\left[\left(\mu^y_{ss}\right)^{\gamma-1}-\beta b_l\right]}\right\} \\ &=\tfrac{M\left[\left(\mu^y_{ss}\right)^{\gamma}-\beta b_c\right]\tau}{\chi\left(\mu^y_{ss}-b_c\right)\left[\left(\mu^y_{ss}\right)^{\gamma-1}-\beta b_l\right]}\end{aligned}\ , \tag{B.12}$$

which allows us to solve for H_{ss}, which is given by

$$H_{ss}=\left\{(1-gy)\left(\frac{\delta_1}{\alpha}\right)^{\alpha/(\alpha-1)}-\frac{M\left[\left(\mu^y_{ss}\right)^{\gamma}-\beta b_c\right]\left[\left(b_l-1\right)\right]}{\chi\left(\mu^y_{ss}-b_c\right)\left[\left(\mu^y_{ss}\right)^{\gamma-1}-\beta b_l\right]}\right\}^{-1}\frac{M\left[\left(\mu^y_{ss}\right)^{\gamma}-\beta b_c\right]\tau}{\chi\left(\mu^y_{ss}-b_c\right)\left[\left(\mu^y_{ss}\right)^{\gamma-1}-\beta b_l\right]}\ . \tag{B.13}$$

Plugging the solution to H_{ss} in equation (B.2), (B.8), (B.9) and (B.10), we can solve for to $\widetilde{K_{ss}}$, $\widetilde{C_{ss}}$, $\widetilde{I_{ss}}$ and $\widetilde{Y_{ss}}$ accordingly. In addition, equation (B.6), combined with $\widetilde{C_{ss}}$ and H_{ss} solves $\widetilde{\Lambda_{ss}}$. Furthermore, given $\widetilde{\Lambda_{ss}}$, $\widetilde{I_{ss}}$ and $\widetilde{Y_{ss}}$, equation (A.11) - (A.13), evaluated at the steady state, would allow us to solve for $\widetilde{P_{ss}}$, R_{ss}^{rf} and $\widetilde{V_{ss}}$.

Till now, the only remaining task is to solve for $\widetilde{G_{ss}}$ and x_{ss}^{g}. From equation (A.14), it is straightforward to show that

$$x_{ss}^{g} = (\mu_{ss}^{y})^{1/(\rho_{xg}-1)} \ . \tag{B.14}$$

Finally, $\widetilde{G_{ss}}$ is solved by using equation (B.11) and the solution to x_{ss}^{g} and $\widetilde{Y_{ss}}$.

C Data Construction

The quarterly data on output growth, investment growth, consumption to output ratio, government expenditures to output ratio, total market values to output ratio, hours and risk-free rate is constructed using the following series:

(1)	Nominal Gross Domestic Product, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 1, billions of dollars seasonally adjusted at annual rate;
(2)	Real Gross Domestic Product, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.6 (Quarterly), line 1, billions of chained 2009 dollars seasonally adjusted at annual rate;
(3)	Nominal Nonresidential Fix Investment, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 9, billions of dollars seasonally adjusted at annual rate;
(4)	Nominal Residential Fix Investment, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 13, billions of dollars seasonally adjusted at annual rate.;
(5)	Implicit Deflator for Fixed Investment, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.9 (Quarterly), line 8, seasonally adjusted;
(6)	Nominal Personal Consumption on Nondurable Goods, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 5, billions of dollars seasonally adjusted at annual rate;
(7)	Nominal Personal Consumption on Services, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 6, billions of dollars seasonally adjusted at annual rate;
(8)	Implicit Deflator for Personal Consumption Expenditure, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.9 (Quarterly), line 2, seasonally adjusted;

(9) Nominal Government Gross Investment, downloaded from Bureau of Economic Analysis (www.bea.gov), National Income and Product Accounts Table 3.9.5 (Quarterly), line 3, billions of dollars, seasonally adjusted at annual rate;

(10) Nominal Government Consumption Expenditure, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 3.9.5 (Quarterly), line 2, billions of dollars, seasonally adjusted at annual rate;

(11) Nonfarm Business Hours Worked, BLS label PRS85006033, downloaded from FRED (research.stlouisfed.org), index 2009=100, seasonally adjusted;

(12) Civilian Non-institutional Population over 16, BLS label LNU00000000Q, downloaded from BLS (www.bls.gov).

(13) 3-Month Treasury Bill Secondary Market Rate (Monthly), downloaded from FRED (research.stlouisfed.org), not seasonally adjusted;

(14) Total Market Values, CRSP data with all the stocks traded on NYSE, AMEX and NASDAQ included, downloaded from WRDS (wrds-web.wharton.upenn.edu/wrds/), thousands of dollars.

The construction of data on Y_t, I_t, A_t, C_t, G_t, M_t, and H_t is straightforward:

$$
\begin{aligned}
\text{GDP Deflator } (GDef_t) &= \quad (1)/(2);\\
\text{Real Per Capita GDP } (Y_t) &= \quad (2)/(12);\\
\text{Real Per Capita Investment } (I_t) &= \quad [(3)+(4)]/(12)/Gdef;\\
\text{Relative Price of Investment } (A_t) &= \quad (5)/(8);\\
\text{Real Per Capita Consumption } (C_t) &= \quad [(6)+(7)]/(12)/Gdef;\\
\text{Real Per Capita Government Expenditure } (G_t) &= \quad [(9)+(10)]/(12)/Gdef;\\
\text{Per Capita Hours Worked } (H_t) &= \quad (5)/(8).
\end{aligned}
$$

In this step, note that Y_t, I_t, A_t, C_t and G_t are scaled by 1,000,000 simply because they are measured in billions of dollars whereas M_t is measured in thousands of dollars.

The construction of the gross real risk free rate takes a few more steps. First, monthly data on 3-month treasury bill rate is converted into quarterly by taking the average of the three observations in each quarter. Second, given that 3-month treasury bill rate is the annualized return measured by percentage points, the data on the quarterly series is divided by 400. In addition, we construct the real risk-free rate by making the assumption of perfect foresight. Based on Fisher's equation, we subtract expected inflation, $log\left(\frac{GDef_{t+1}}{GDef_t}\right)$, and add 1 to obtain the gross real risk-free rate.

D Additional Tables

Table 6: Bayesian Estimation of Structural Parameters : Specification 2

	Prior			Posterior				
Parameter	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	1.3758	1.3767	0.1106	1.3542	1.3956
κ	Gamma	4	2	7.0746	7.0769	0.0367	7.0556	7.0889
b_c	Beta	0.5	0.2	0.7377	0.7376	0.0151	0.7346	0.7417
b_l	Beta	0.5	0.2	0.8931	0.8933	0.0235	0.8895	0.8960
χ	Gamma	4	2	4.5771	4.5756	0.1642	4.5576	4.5993
δ_2	Gamma	0.1	0.05	0.1259	0.1260	0.0012	0.1254	0.1263
$\phi_1^{x,T}$	Beta	0.6	0.3	0.1543	0.1523	0.0248	0.1443	0.1660
$\phi_2^{x,T}$	Beta	0.2	0.1	0.1309	0.1308	0.0062	0.1286	0.1332
$\phi_1^{a,T}$	Beta	0.6	0.3	0.6906	0.6915	0.0048	0.6838	0.6974
$\phi_2^{a,T}$	Beta	0.2	0.1	0.2504	0.2506	0.0060	0.2461	0.2549
$\phi_1^{z,T}$	Beta	0.6	0.3	0.3099	0.3116	0.0106	0.3032	0.3154
$\phi_2^{z,T}$	Beta	0.2	0.1	0.1836	0.1838	0.0086	0.1814	0.1851
ϕ_1^{ξ}	Beta	0.6	0.3	0.6549	0.6546	0.0222	0.6532	0.6568
ϕ_2^{ξ}	Beta	0.2	0.1	0.3448	0.3452	0.0090	0.3430	0.3466
$\phi_1^{z^I}$	Beta	0.6	0.3	0.0870	0.0874	0.0339	0.0828	0.0908
$\phi_2^{z^I}$	Beta	0.2	0.1	0.2339	0.2343	0.0051	0.2314	0.2358
ϕ_1^{g}	Beta	0.6	0.3	0.4955	0.4958	0.0096	0.4921	0.4985
ϕ_2^{g}	Beta	0.2	0.1	0.3000	0.2995	0.0050	0.2974	0.3031
ρ^{xg}	Beta	0.7	0.3	0.5745	0.5740	0.0071	0.5684	0.5817
$\sigma_{x,P}$	/	/	/	/	/	/	/	/
$\sigma_{x,T}$	Inverse-Gamma	0.1	Inf.	0.0202	0.0201	0.0024	0.0193	0.0213
$\sigma_{a,P}$	Inverse-Gamma	0.02	Inf.	0.0135	0.0134	0.0015	0.0285	0.0322
$\sigma_{a,T}$	Inverse-Gamma	0.1	Inf.	0.0306	0.0306	0.0015	0.0285	0.0322
$\sigma_{z,P}$	Inverse-Gamma	0.02	Inf.	0.0247	0.0247	0.0013	0.0235	0.0258
$\sigma_{z,T}$	Inverse-Gamma	0.1	Inf.	0.0174	0.0173	0.0058	0.0164	0.0185
σ_{ξ}	Inverse-Gamma	0.1	Inf.	0.0449	0.0449	0.1867	0.0418	0.0481
σ_{z^I}	Inverse-Gamma	0.1	Inf.	0.2037	0.2049	0.0098	0.1915	0.2144
σ_g	Inverse-Gamma	0.1	Inf.	0.0345	0.0346	0.0020	0.0326	0.0364

Table 7: Bayesian Estimation of Structural Parameters : Specification 3

	Prior			Posterior				
Parameter	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	2.2855	2.2860	0.0253	2.2812	2.2891
κ	Gamma	4	2	5.6500	5.6499	0.0067	5.6405	5.6604
b_c	Beta	0.5	0.2	0.5324	0.5324	0.0014	0.5320	0.5329
b_l	Beta	0.5	0.2	0.6949	0.6949	0.0021	0.6944	0.6955
χ	Gamma	4	2	2.5873	2.5860	0.0102	2.5826	2.5934
δ_2	Gamma	0.1	0.05	0.0801	0.0801	0.0002	0.800	0.0801
$\phi_1^{x,T}$	Beta	0.6	0.3	0.5510	0.5509	0.0063	0.5504	0.5516
$\phi_2^{x,T}$	Beta	0.2	0.1	0.2990	0.2990	0.0017	0.2988	0.2993
$\phi_1^{a,T}$	Beta	0.6	0.3	0.3462	0.3462	0.0015	0.3456	0.3466
$\phi_2^{a,T}$	Beta	0.2	0.1	0.1491	0.1492	0.0009	0.1486	0.1494
$\phi_1^{z,T}$	Beta	0.6	0.3	0.7377	0.7378	0.0018	0.7360	0.7388
$\phi_2^{z,T}$	Beta	0.2	0.1	0.1306	0.1306	0.0012	0.1305	0.1308
ϕ_1^{ξ}	Beta	0.6	0.3	0.4461	0.4461	0.0088	0.4446	0.4474
ϕ_2^{ξ}	Beta	0.2	0.1	0.2181	0.2181	0.0014	0.2179	0.2182
$\phi_1^{z^I}$	Beta	0.6	0.3	0.6521	0.6520	0.0022	0.6516	0.6527
$\phi_2^{z^I}$	Beta	0.2	0.1	0.3431	0.3431	0.0014	0.3428	0.3433
ϕ_1^{g}	Beta	0.6	0.3	0.6905	0.6905	0.0120	0.6903	0.6906
ϕ_2^{g}	Beta	0.2	0.1	0.3094	0.3094	0.0011	0.3093	0.3095
ρ^{xg}	Beta	0.7	0.3	0.8524	0.8526	0.0047	0.8511	0.8535
$\sigma_{x,P}$	Inverse-Gamma	0.02	Inf.	0.0082	0.0082	0.0107	0.0077	0.0087
$\sigma_{x,T}$	Inverse-Gamma	0.1	Inf.	0.0387	0.0383	0.0027	0.0372	0.0408
$\sigma_{a,P}$	/	/	/	/	/	/	/	/
$\sigma_{a,T}$	Inverse-Gamma	0.1	Inf.	0.0344	0.0344	0.0017	0.0336	0.0353
$\sigma_{z,P}$	Inverse-Gamma	0.02	Inf.	0.0251	0.0251	0.0014	0.0246	0.0257
$\sigma_{z,T}$	Inverse-Gamma	0.1	Inf.	0.0274	0.0274	0.0012	0.0268	0.0278
σ_{ξ}	Inverse-Gamma	0.1	Inf.	0.1013	0.1009	0.0051	0.0970	0.1070
σ_{z^I}	Inverse-Gamma	0.1	Inf.	0.1450	0.1457	0.0072	0.1410	0.1474
σ_g	Inverse-Gamma	0.1	Inf.	0.0201	0.0200	0.0209	0.0187	0.0217

Table 8: Bayesian Estimation of Structural Parameters : Specification 4

	Prior			Posterior				
Parameter	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	1.3097	1.3067	0.0016	1.2861	1.3380
κ	Gamma	4	2	3.3802	3.3712	0.0069	3.3376	3.4396
b_c	Beta	0.5	0.2	0.2196	0.2187	0.0004	0.2115	0.2292
b_l	Beta	0.5	0.2	0.2627	0.2640	0.0005	0.2549	0.2694
χ	Gamma	4	2	4.9082	4.9025	0.0055	4.8614	4.9652
δ_2	Gamma	0.1	0.05	0.0374	0.0374	0.0001	0.0332	0.0409
$\phi_1^{x,T}$	Beta	0.6	0.3	0.3014	0.3029	0.0007	0.2908	0.3099
$\phi_2^{x,T}$	Beta	0.2	0.1	0.2689	0.2667	0.0003	0.2586	0.2834
$\phi_1^{a,T}$	Beta	0.6	0.3	0.6842	0.6839	0.0005	0.6812	0.6874
$\phi_2^{a,T}$	Beta	0.2	0.1	0.2635	0.2635	0.0003	0.2620	0.2652
$\phi_1^{z,T}$	Beta	0.6	0.3	0.4784	0.4782	0.0003	0.4747	0.4825
$\phi_2^{z,T}$	Beta	0.2	0.1	0.2420	0.2417	0.0002	0.2392	0.2450
ϕ_1^{ξ}	Beta	0.6	0.3	0.8239	0.8239	0.0010	0.8207	0.8267
ϕ_2^{ξ}	Beta	0.2	0.1	0.1670	0.1668	0.0002	0.1646	0.1702
$\phi_1^{z^I}$	Beta	0.6	0.3	0.5932	0.5931	0.0008	0.5805	0.6062
$\phi_2^{z^I}$	Beta	0.2	0.1	0.2063	0.2033	0.0002	0.1975	0.2191
ϕ_1^{g}	Beta	0.6	0.3	0.6162	0.6145	0.0006	0.6044	0.6334
ϕ_2^{g}	Beta	0.2	0.1	0.1502	0.1512	0.0002	0.1455	0.1535
ρ^{xg}	Beta	0.7	0.3	0.1578	0.1621	0.0015	0.1373	0.1731
$\sigma_{x,P}$	/	/	/	/	/	/	/	/
$\sigma_{x,T}$	Inverse-Gamma	0.1	Inf.	0.0341	0.0340	0.0019	0.0322	0.0361
$\sigma_{a,P}$	/	/	/	/	/	/	/	/
$\sigma_{a,T}$	Inverse-Gamma	0.1	Inf.	0.0133	0.0133	0.0003	0.0124	0.0143
$\sigma_{z,P}$	Inverse-Gamma	0.02	Inf.	0.0290	0.0289	0.0008	0.0271	0.0309
$\sigma_{z,T}$	Inverse-Gamma	0.1	Inf.	0.0168	0.0168	0.0016	0.0158	0.0180
σ_{ξ}	Inverse-Gamma	0.1	Inf.	0.0516	0.0515	0.0170	0.0484	0.0550
σ_{z^I}	Inverse-Gamma	0.1	Inf.	0.1456	0.1455	0.0279	0.1321	0.1592
σ_g	Inverse-Gamma	0.1	Inf.	0.0466	0.0465	0.0012	0.0434	0.0500

Table 9: Unconditional Variance Decomposition : Specification 2

	Shock	g^Y	g^{AI}	$log\left(\frac{C}{Y}\right)$	$log\left(\frac{G}{Y}\right)$	$log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	/	/	/	/	/	/	/
	$\epsilon^{a,P}$	0.4587	0.3894	0.2998	0.4440	0.7267	0.6304	0.1084
	$\epsilon^{z,P}$	0.0067	0.0251	0.0053	0.0040	0.0016	0.0157	0.1926
Sum		0.4654	0.4145	0.3051	0.4480	0.7283	0.6461	0.3031
Transitory	$\epsilon^{x,T}$	0.0507	0.0117	0.0005	0.0017	0.0004	0.0006	0.0113
	$\epsilon^{a,T}$	0.4614	0.5313	0.1675	0.3435	0.2698	0.3275	0.1044
	$\epsilon^{z,T}$	0.0028	0.0111	0.0000	0.0001	0.0008	0.0000	0.5101
	ϵ^{ξ}	0.0004	0.0001	0.5267	0.1996	0.0004	0.0255	0.0013
	ϵ^{z^I}	0.0070	0.0297	0.0001	0.0006	0.0002	0.0001	0.0154
	ϵ^{g}	0.0123	0.0015	0.0000	0.0063	0.0001	0.0000	0.0566
Sum		0.5346	0.5854	0.6948	0.5518	0.2717	0.3537	0.6991

Table 10: Unconditional Variance Decomposition : Specification 3

	Shock	g^Y	g^{AI}	$log\left(\frac{C}{Y}\right)$	$log\left(\frac{G}{Y}\right)$	$log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	0.5854	0.6343	0.5125	0.1277	0.8770	0.5601	0.5418
	$\epsilon^{a,P}$	/	/	/	/	/	/	/
	$\epsilon^{z,P}$	0.0031	0.0050	0.0048	0.0055	0.0009	0.0301	0.0668
Sum		0.5885	0.6393	0.5173	0.1332	0.8779	0.5902	0.6086
Transitory	$\epsilon^{x,T}$	0.3681	0.2856	0.1511	0.0720	0.0952	0.1247	0.1743
	$\epsilon^{a,T}$	0.0180	0.0221	0.0170	0.0066	0.0095	0.0160	0.0070
	$\epsilon^{z,T}$	0.0069	0.0148	0.0010	0.0019	0.0009	0.0011	0.1822
	ϵ^{ξ}	0.0013	0.0022	0.0026	0.0000	0.0002	0.0001	0.0149
	ϵ^{z^I}	0.0142	0.0358	0.0482	0.0322	0.0164	0.1566	0.0087
	ϵ^{g}	0.0023	0.0001	0.2628	0.7541	0.0000	0.1113	0.0042
Sum		0.4116	0.3606	0.4827	0.8668	0.1222	0.4098	0.3913

Table 11: Unconditional Variance Decomposition : Specification 4

	Shock	g^Y	g^{AI}	$log\left(\frac{C}{Y}\right)$	$log\left(\frac{G}{Y}\right)$	$log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	/	/	/	/	/	/	/
	$\epsilon^{a,P}$	/	/	/	/	/	/	/
	$\epsilon^{z,P}$	0.3723	0.0633	0.0524	0.5616	0.0095	0.0829	0.1035
Sum		/	/	/	/	/	/	/
Transitory	$\epsilon^{x,T}$	0.2770	0.0769	0.0474	0.0061	0.0298	0.0077	0.1243
	$\epsilon^{a,T}$	0.0873	0.4429	0.7889	0.1937	0.9332	0.1153	0.1746
	$\epsilon^{z,T}$	0.1049	0.0115	0.0012	0.0037	0.0002	0.0008	0.3958
	ϵ^{ξ}	0.0821	0.0216	0.0212	0.1935	0.0142	0.7667	0.1048
	ϵ^{z^I}	0.0647	0.3828	0.0867	0.0238	0.0129	0.0261	0.0624
	ϵ^{g}	0.0116	0.0011	0.0022	0.0176	0.0002	0.0005	0.0346
Sum		0.6276	0.9368	0.9476	0.4384	0.9905	0.9171	0.8965

E Supplemental Figures to Conditional Variance Decomposition

Figure E.1: Conditional Variance Decomposition : Baseline

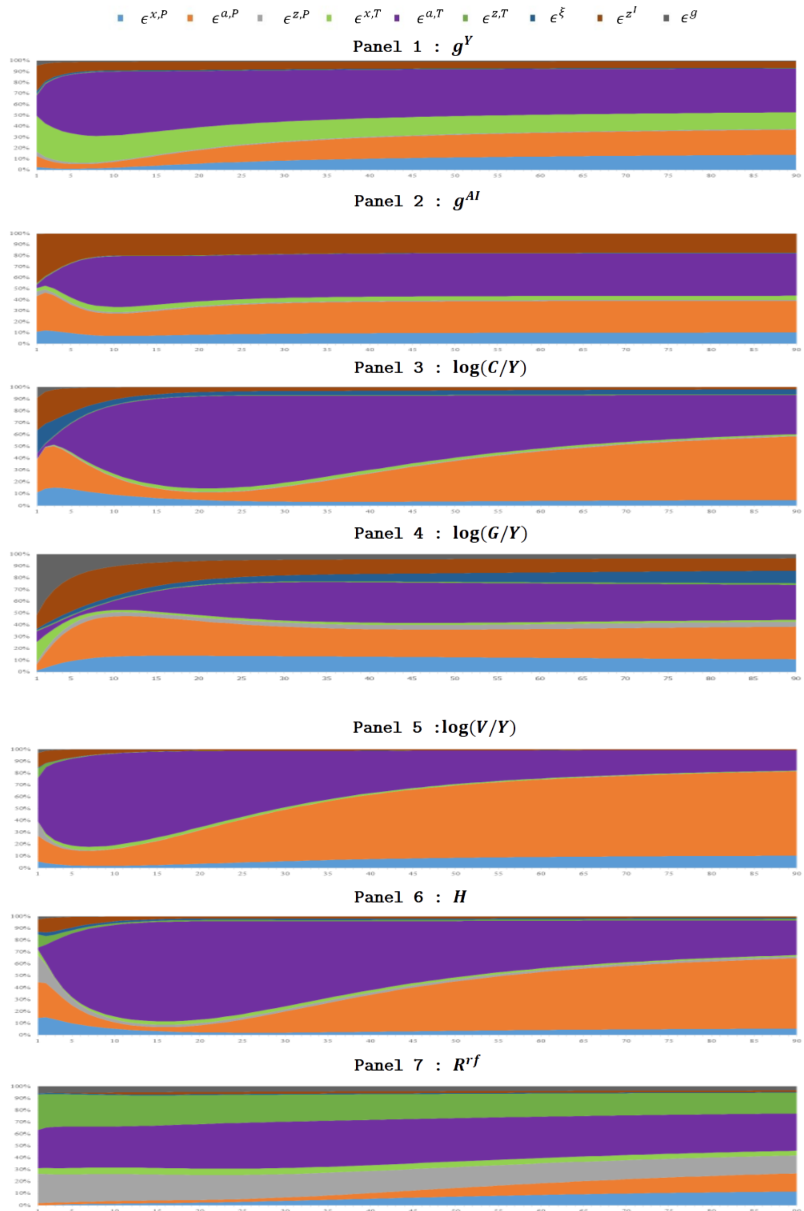


Figure E.2: Conditional Variance Decomposition : Specification 2

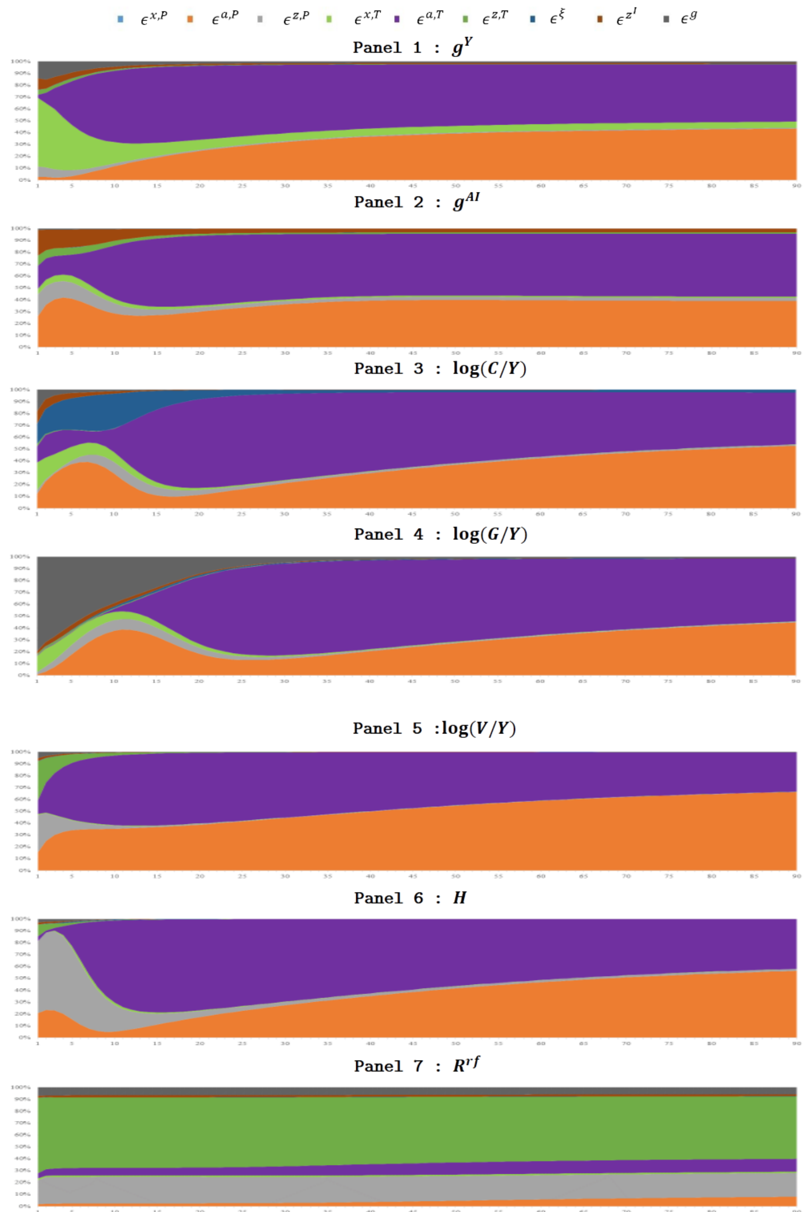


Figure E.3: Conditional Variance Decomposition : Specification 3

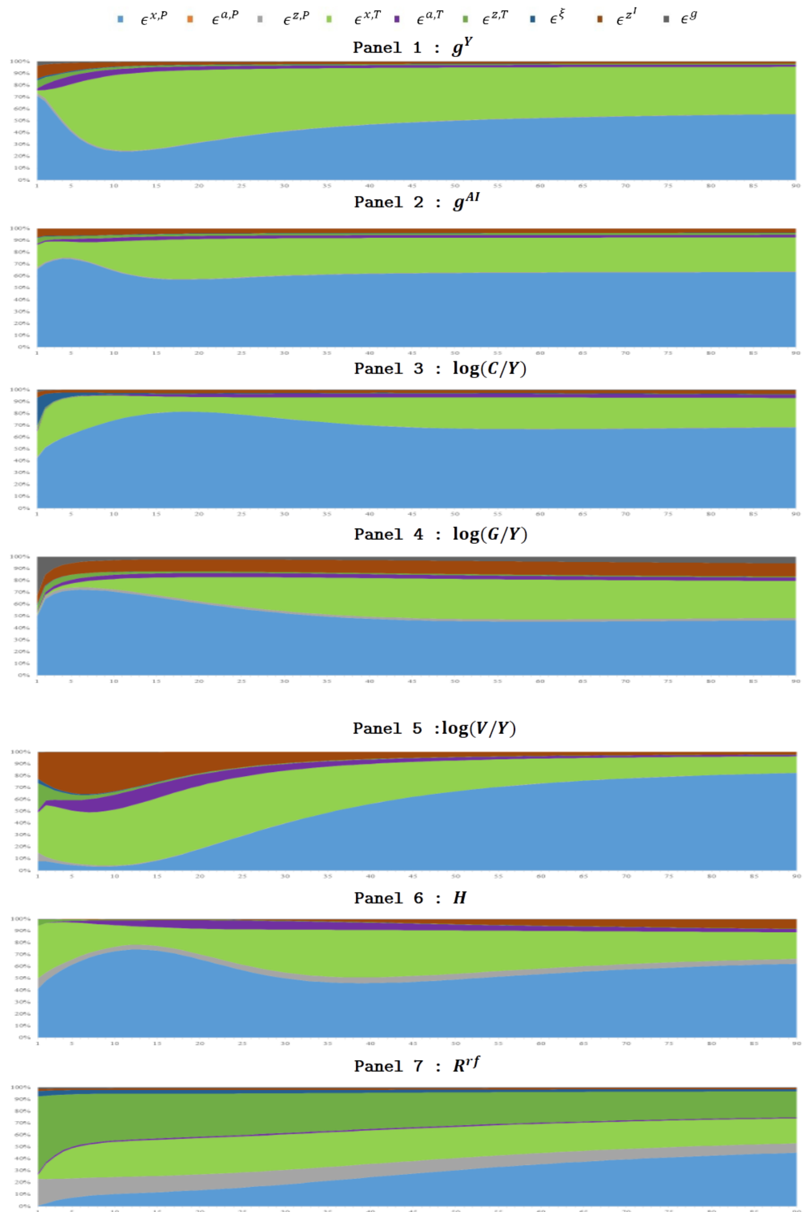
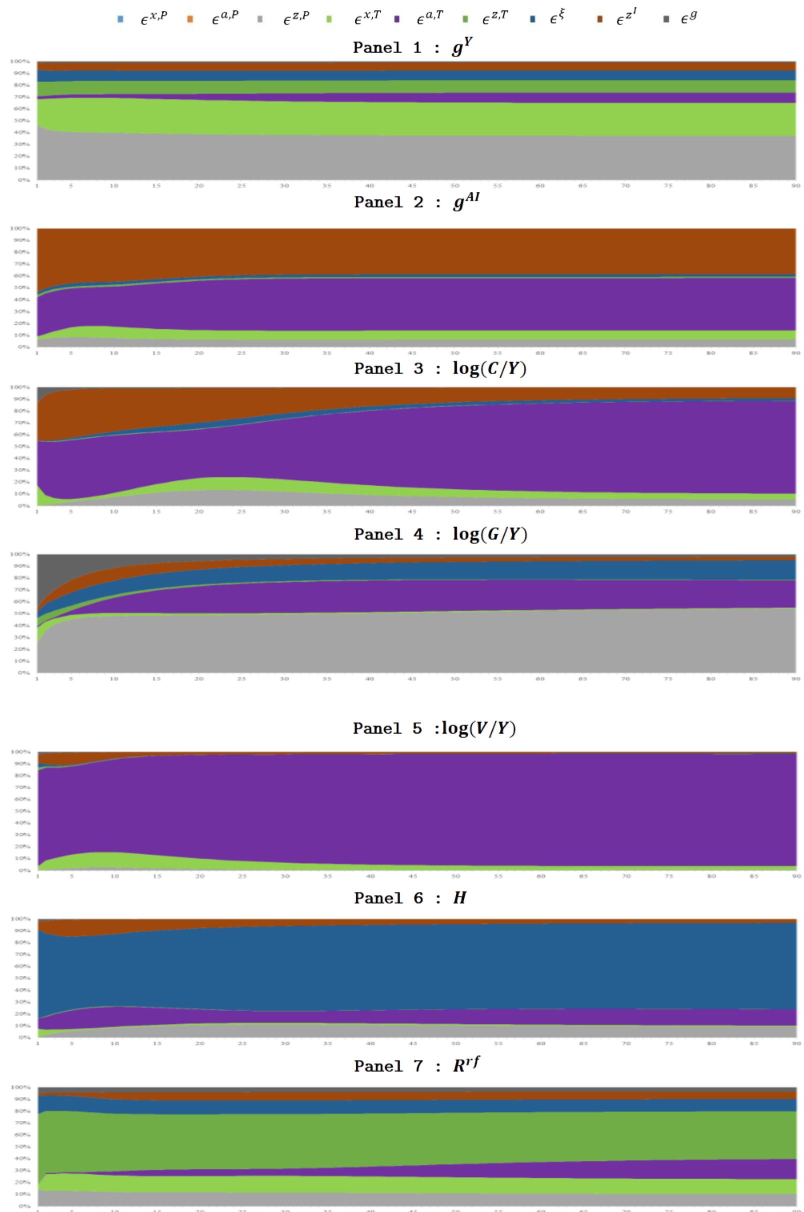


Figure E.4: Conditional Variance Decomposition : Specification 4



F Model Fit and Smoothed Variables

Figure F.1: Actual and Smoothed Variables : Baseline

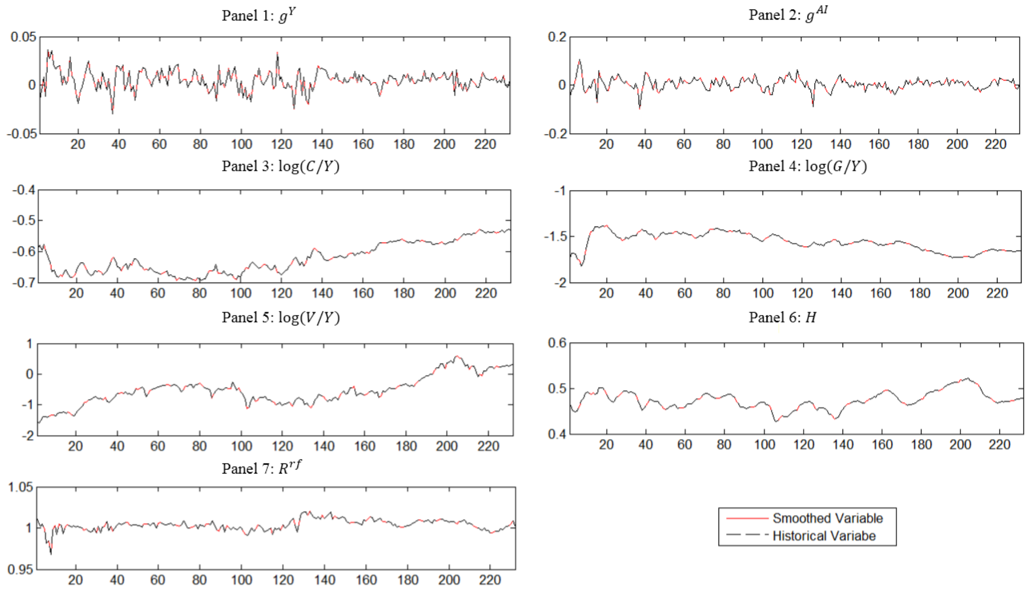


Figure F.2: Smoothed Shocks : Baseline

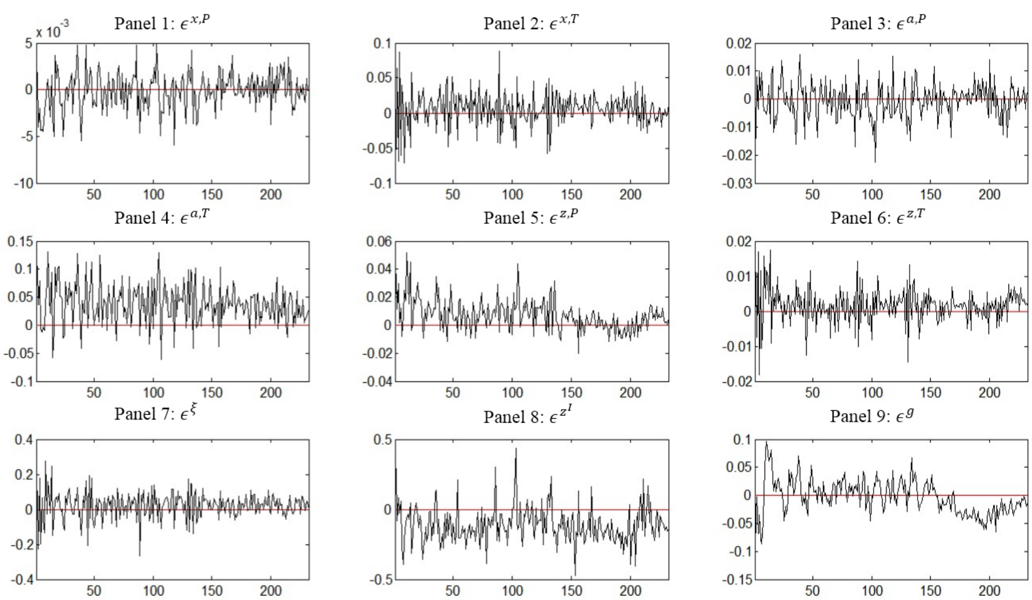


Figure F.3: Actual and Smoothed Variables : Specification 2

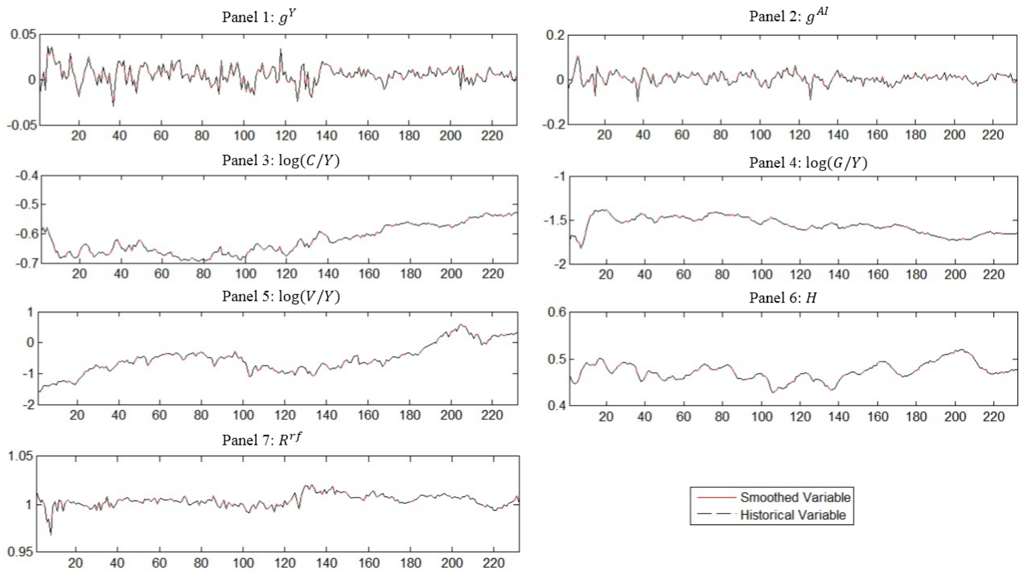


Figure F.4: Smoothed Shocks : Specification 2

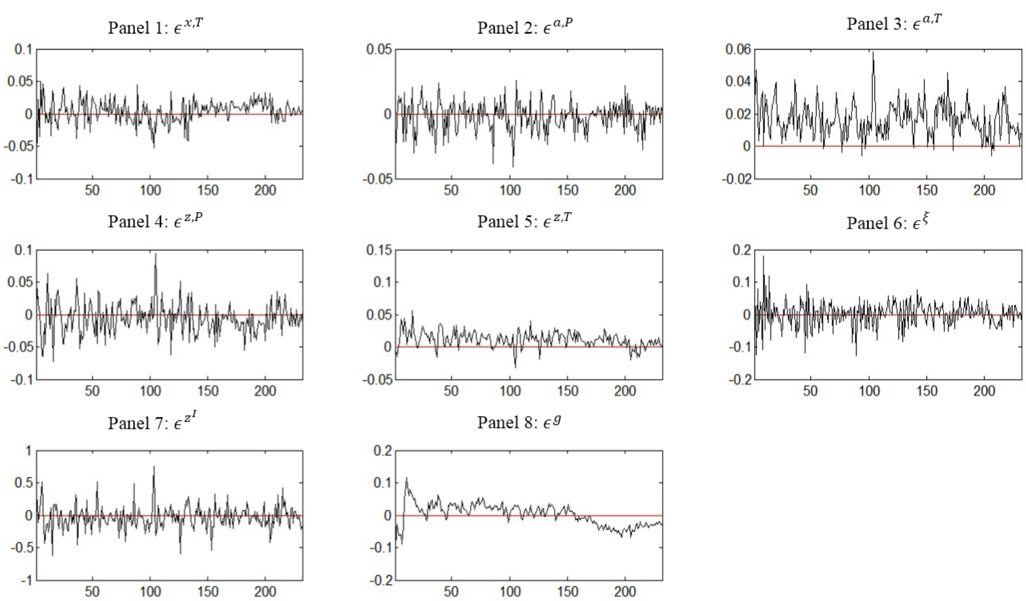


Figure F.5: Actual and Smoothed Variables : Specification 3

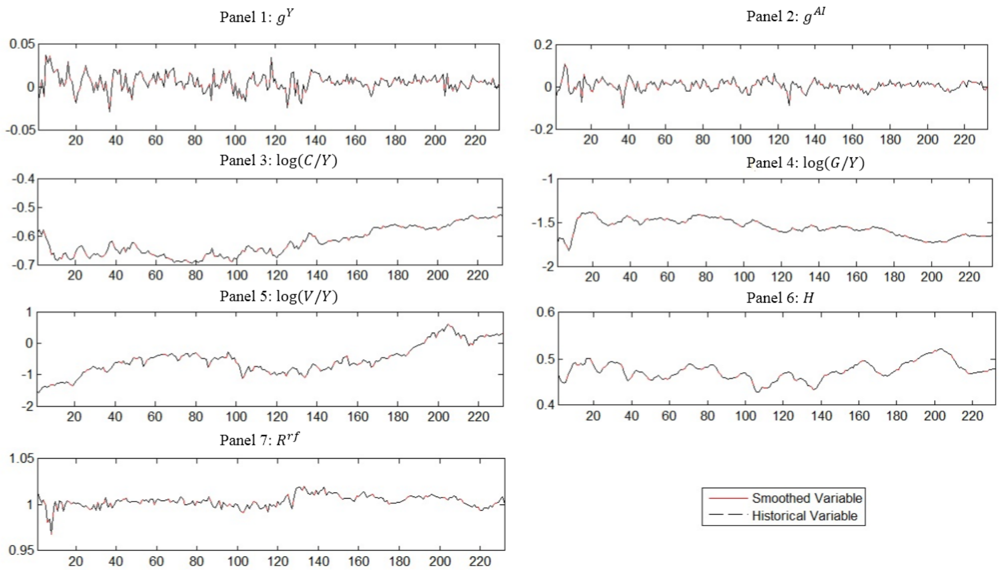


Figure F.6: Smoothed Shocks : Specification 3

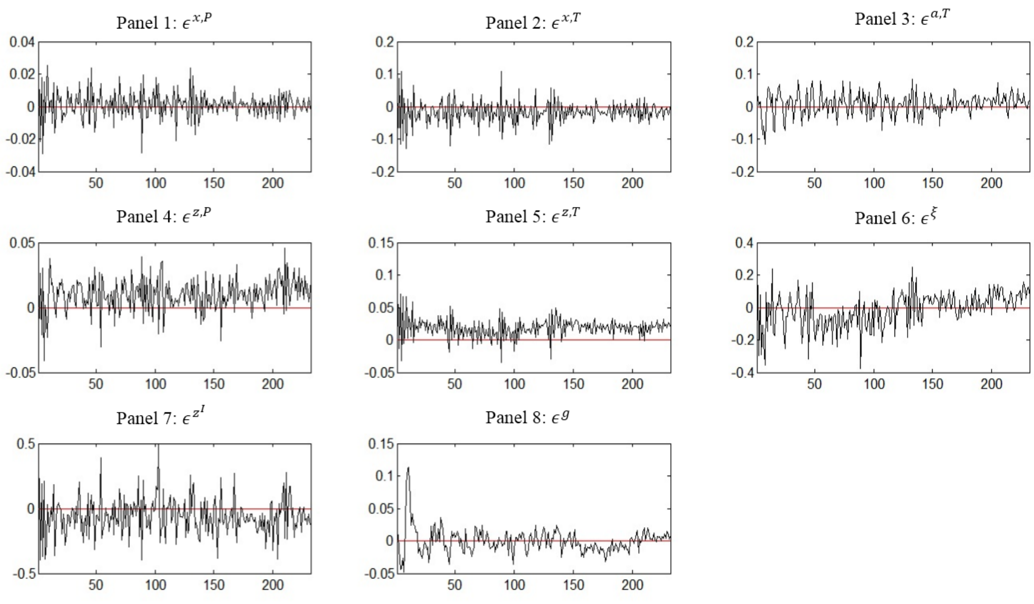


Figure F.7: Actual and Smoothed Variables : Specification 4

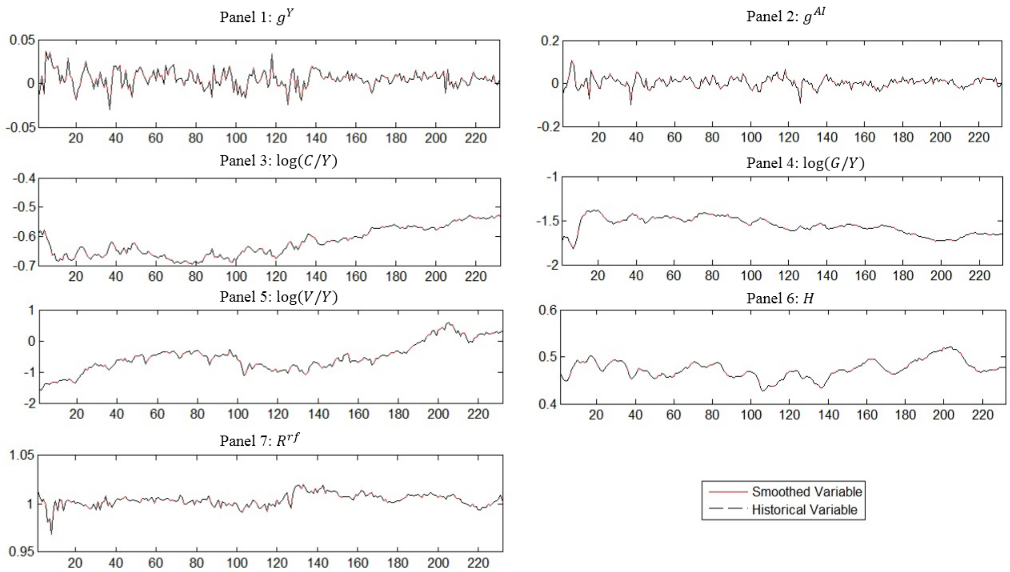
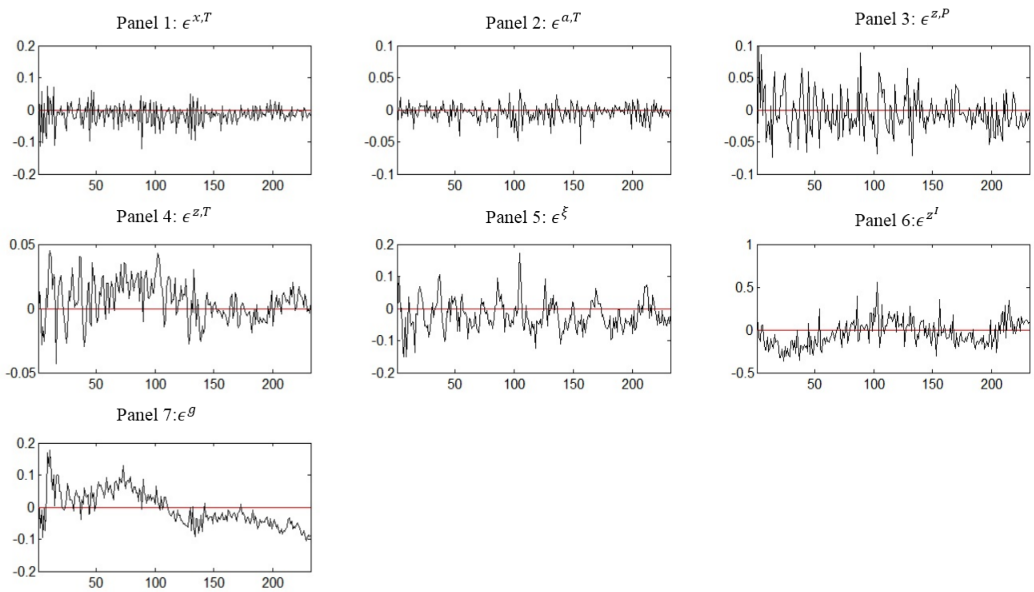


Figure F.8: Smoothed Shocks : Specification 4



G Supplemental Figures to Impulse Responses Analysis

Figure G.1: Dynamic Responses to $\epsilon^{a.P}$

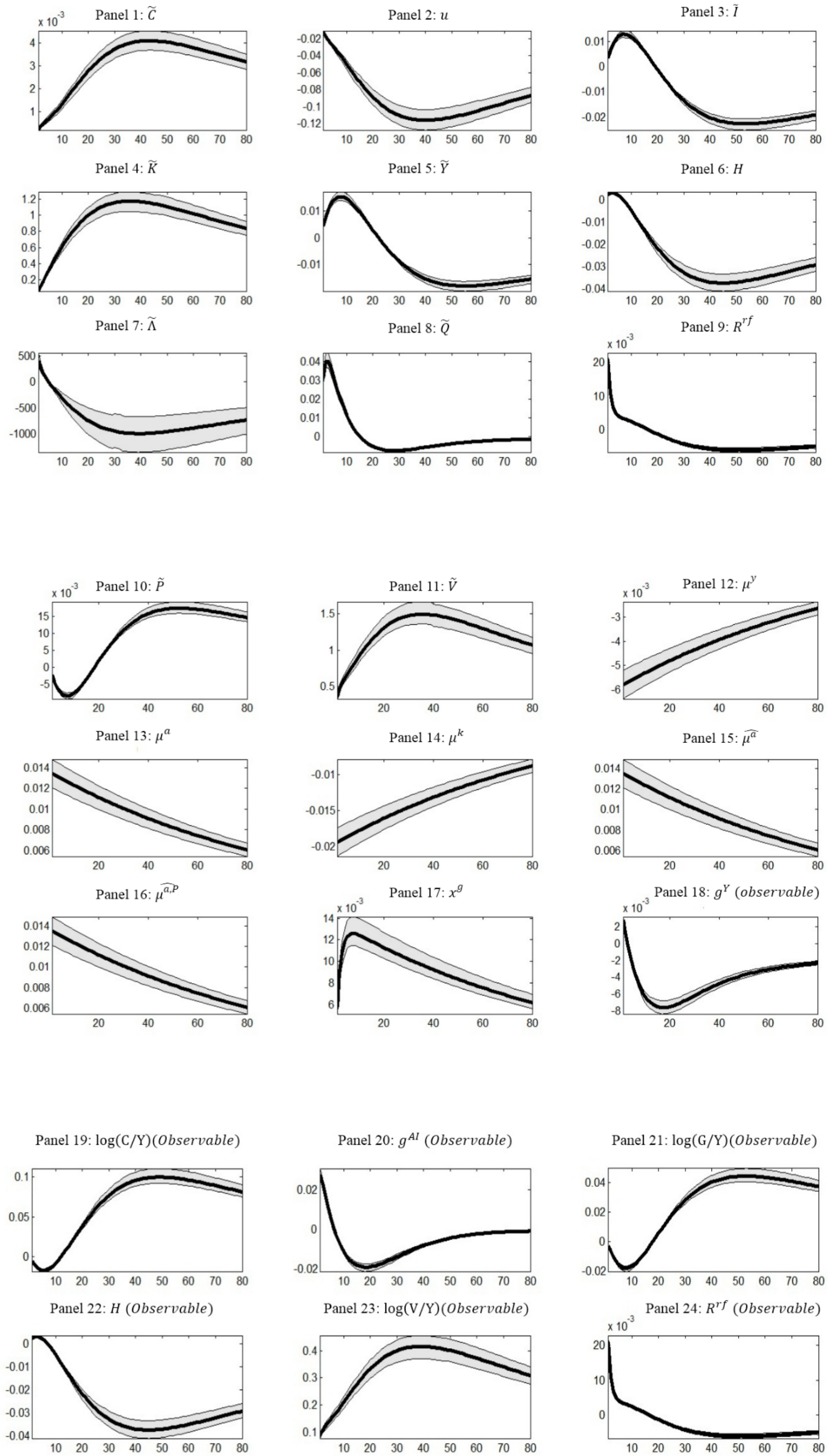


Figure G.2: Dynamic Responses to $\epsilon^{a.T}$

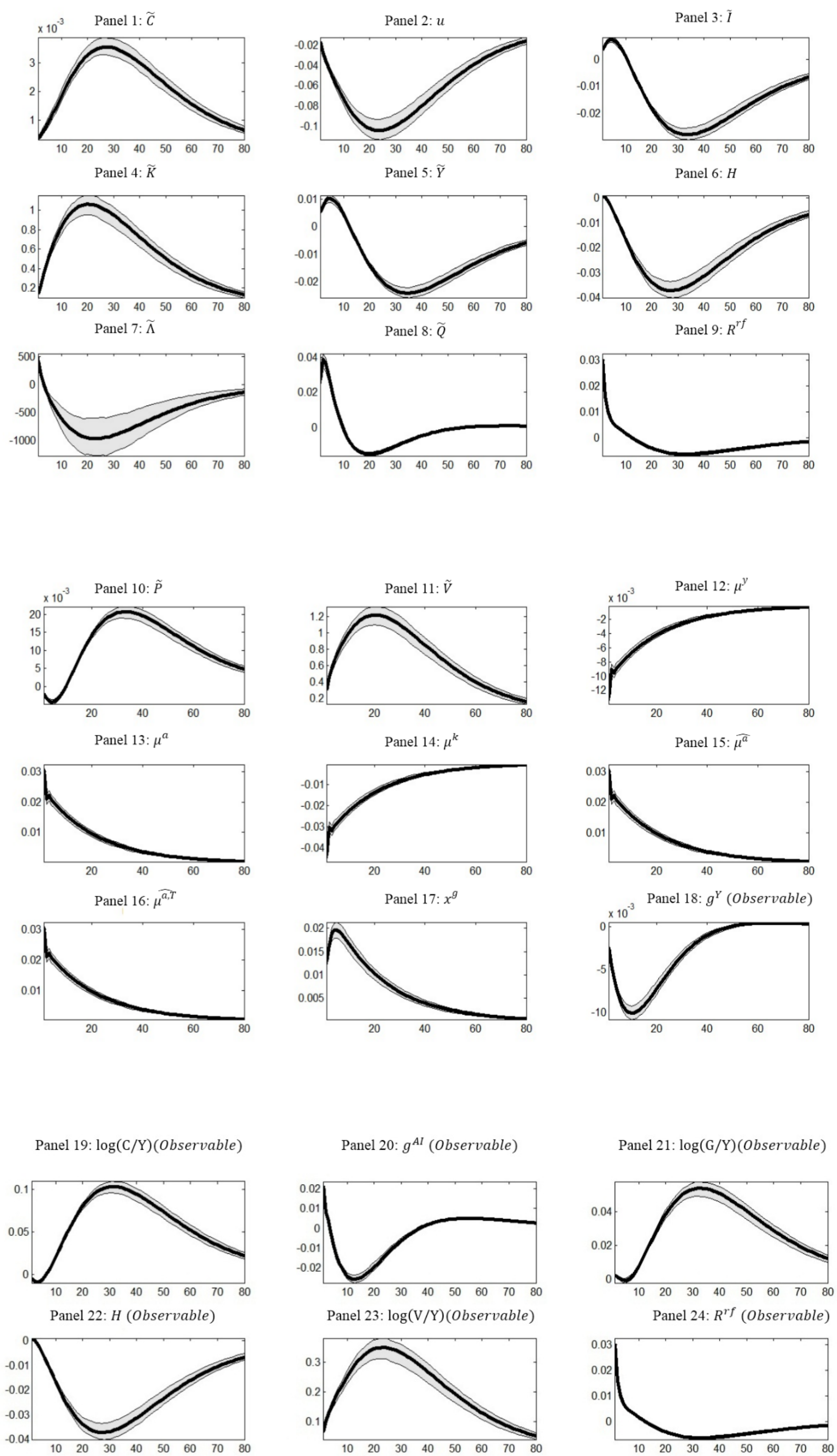


Figure G.3: Dynamic Responses to $\epsilon^{z.P}$

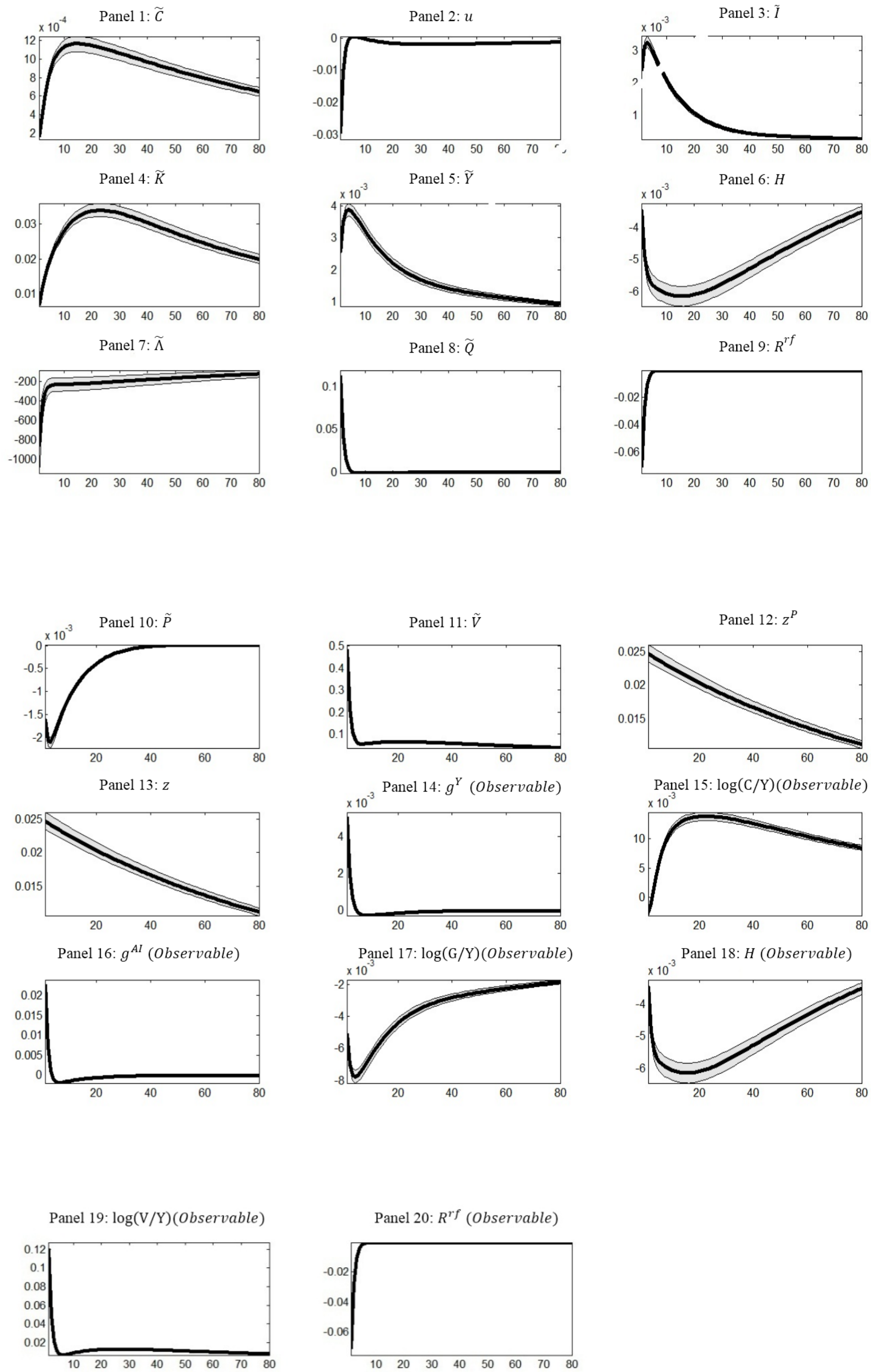


Figure G.4: Dynamic Responses to $\epsilon^{z.T}$

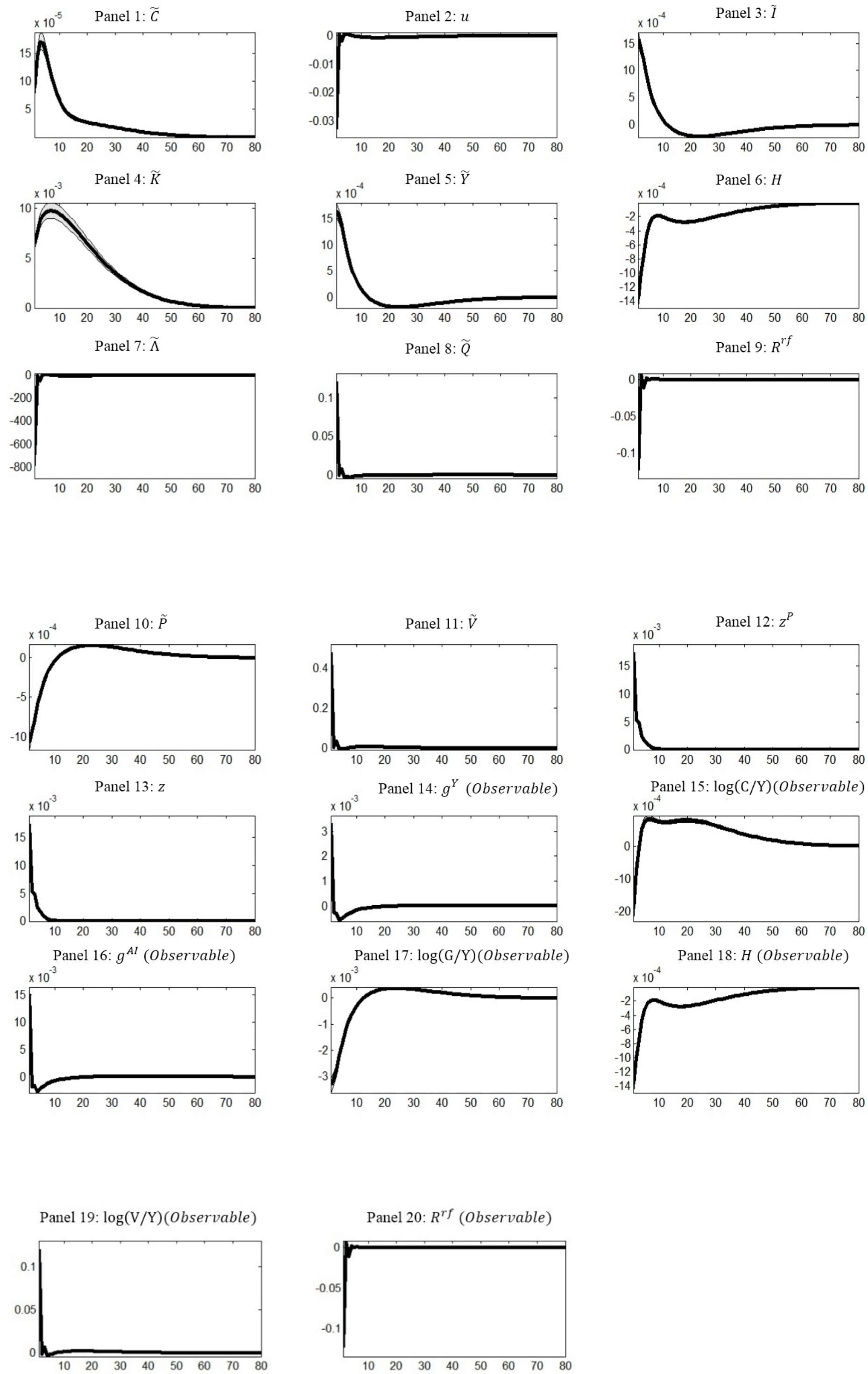


Figure G.5: Dynamic Responses to $\epsilon^{x.T}$

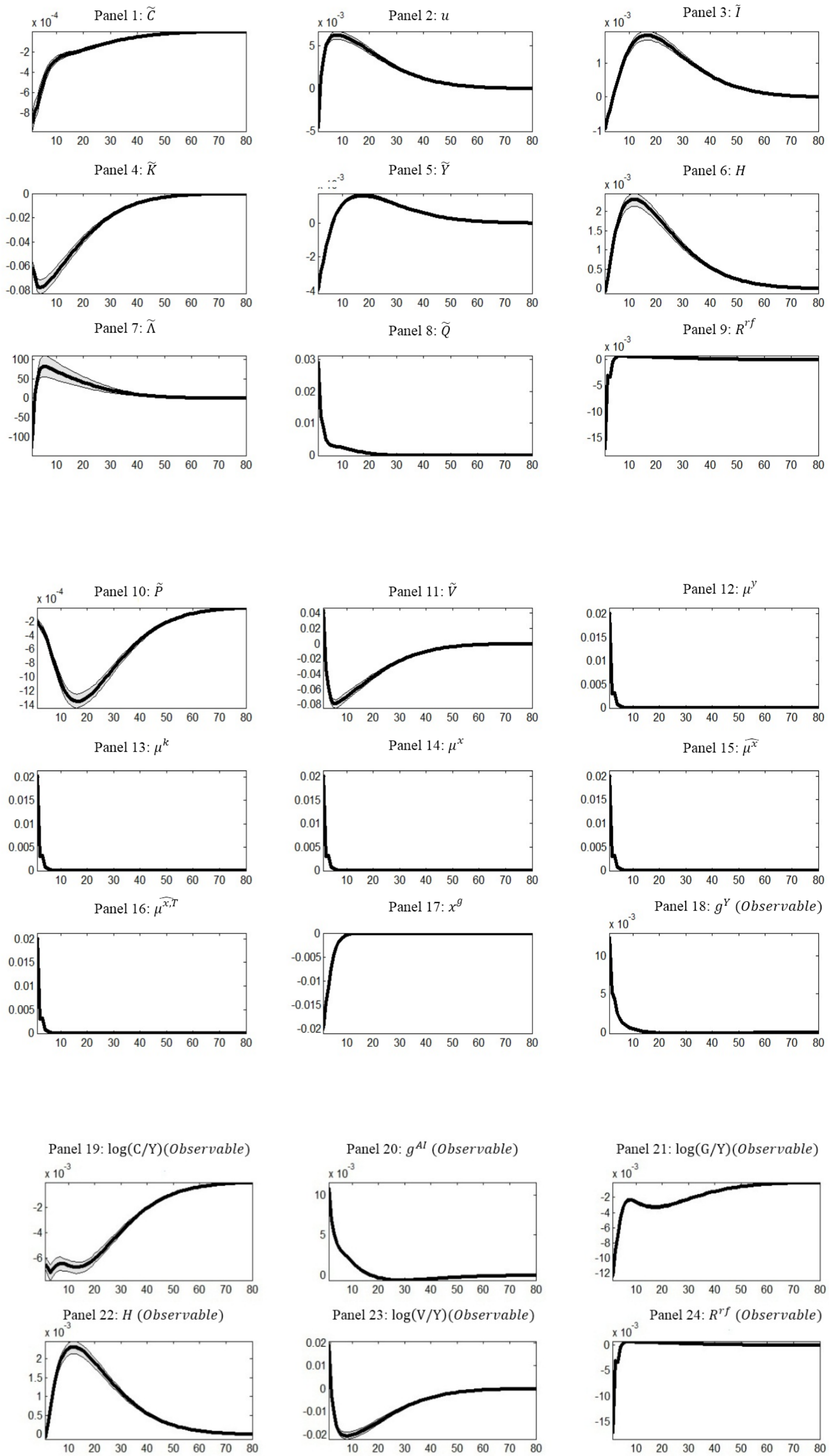


Figure G.6: Dynamic Responses to ϵ^{ξ}

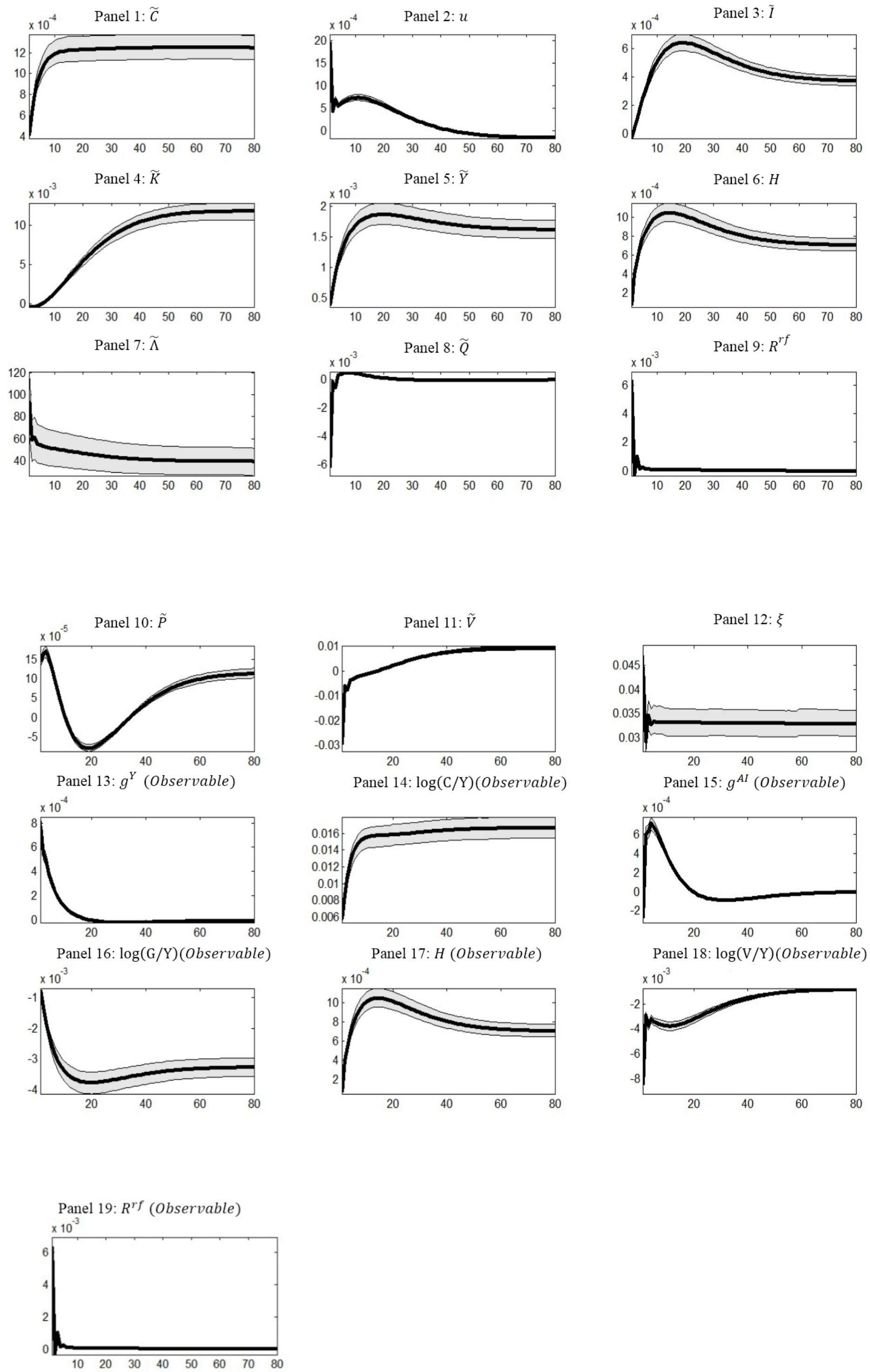


Figure G.7: Dynamic Responses to ϵ^{z^I}

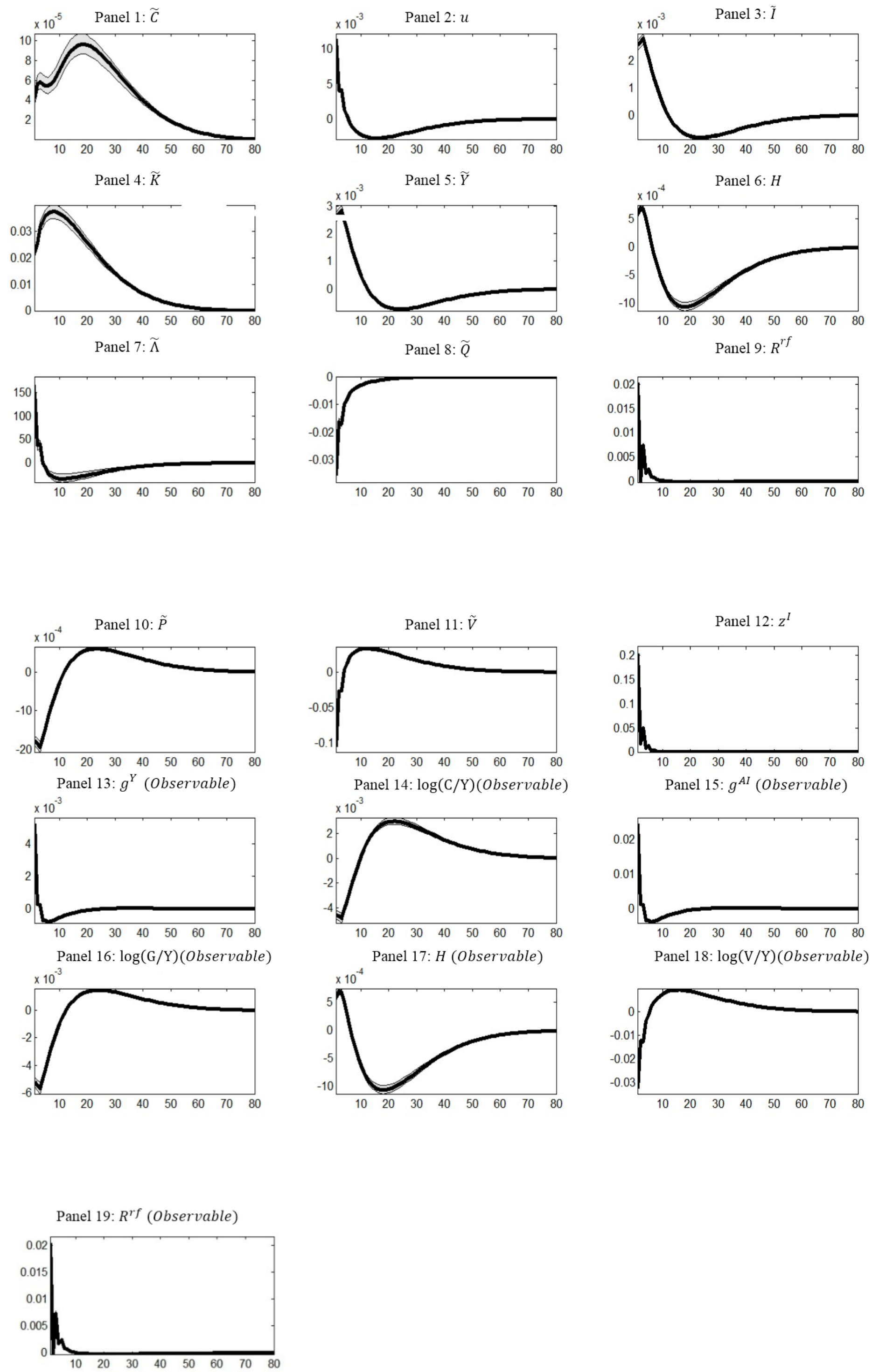


Figure G.8: Dynamic Responses to ϵ^g